Fast and accurate implementation of radial k-t FOCUSS for dynamic MRI using implicit gridding

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Introduction : For dynamic MR imaging , radial trajectory is often employed since it is more robust to motion artifacts due to the over-sampled k-space center region. Furthermore, scan time can be further reduced using view undersampling. However, this view undersampling often causes visually annoying streaking artifacts. In radial k-t FOCUSS [1], high spatio-temporal cine imaging without streaking artifact were successfully obtained from undersampled radial trajectory using FOCUSS(FOCal Underdetermined System Solver) [2] that exploits the sparsity of fact x-f support of cardiac cine. We further showed that radial k-t BLAST/SENSE is an approximation of our radial k-t FOCUSS that is asymptotically optimal from compressed sensing perspective. However, the radial k-t FOCUSS was computationally expensive due to the iterative application of the projection and backprojection steps. Application of gridding algorithm to reduce the computational complexity was, however, not often successful due to the propagation of gridding artifacts during FOCUSS iteration. The main contribution of this paper is to eliminate the necessity of gridding or backprojection/reprojection steps by showing that the non-regular sampling structure can be easily incorporated as a filtering process during k-t FOCUSS iteration in regular grid. This implicit gridding operation incurs only minimial computational overhead. Experimental results demonstrate that the modified radial k-t FOCUSS significantly improves the radial k-t FOCUSS algorithm both in reconstruction speed as well as image quality.

Theory : Instead of directly minimizing L1 cost function, FOCUSS performs L1 minimization using reweighed norm. In cardiac image, signal of x-f domain is sparse because of periodic motion. Radial k-t FOCUSS [1] exploits the sparsity of the x-f support using the FOCUSS. However, one of the drawbacks of the radial k-t FOCUSS is the computational complexity due to the iterative application of projection/backprojection. The main contribution of this paper is to demonstrate that this computational complexity can be significantly reduced by exploit the sampling structure in the radial acquisition. Specifically, the forward model for radial trajectory can be represented by $y = AF_3 m$ where y is the data vector acquired from the radial trajectory along k-t space, F_3 is three dimensional Fourier transform operator on a regular sampling grid with respect to x-y-f signal m , and A is a generalized sampling operator from rectangular grid to the radial trajectory, respectively. The generalized sampling operator can be linearly approximated by $f(x_i) \leftarrow w_i f(x_0)$ and $w_1 + w_2 + w_3 + w_4 = 1$. Now consider the following reweighed least square problem: $C(q) = \left\| y - AF_3 W q \right\|^2 + \lambda \left\| Wq \right\|^2$ where $m = Wq$. The cost function can be minimized using nonlinear optimization techniques where the gradient can be calculated as following $g = -F_3^H A^H (y - AF_3 Wq) + \lambda Wq$

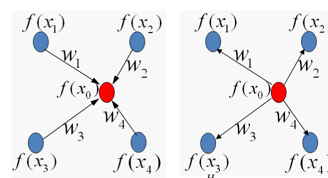
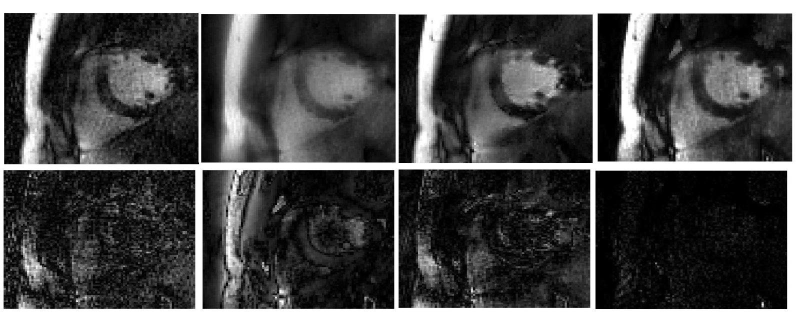


Fig.1 diagram of A and A^H matrix

where A^H denotes the adjoint operator between the radial to rectangular grid. The adjoint operator, A^H can be represented in pointwise as $f(x_0) \leftarrow w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) + w_4 f(x_4)$. Note that this implicit gridding during FOCUSS iteration does not need explicit griddig or projection operator. The novelty of k-t FOCUSS comes from that the weighting matrix W can be continuously updated using the previous solution. (hence, $\Theta = WW^H$ is updated accordingly)[1].



(a)Direct Inversion (b)Radial k-t SENSE (c)Radial k-t FOCUSS (d) Modified radial k-t FOCUSS

Result : In order to evaluate the performance of the modified radial k-t FOCUSS, we apply this method for real data. In this experiment, we have acquired 12 frames of full radial k-space cardiac sequence and applied view undersampling by reduction of two. Fig.2 illustrates reconstructed image of direct inversion, radial k-t SENSE[4], radial k-t FOCUSS[1], and the modified k-t FOCUSS using implicit gridding. As illustrated in Fig.2, significant improvement of reconstruction quality was observed even though the complexity is reduced by factor of six

Conclusion : We proposed the radial k-t FOCUSS algorithm to obtain sparse solutions from angularly undersampled k-t data. The new algorithm did not require the explicit gridding or time consuming backprojection/reprojection. Furthermore, reconstructed image quality was significantly improved.

Reference

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