Data Interpolation in Phase-scrambling Fourier Transform Technique by Modified Gerchberg's Algorithm for Alias-free Image Reconstruction

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Introduction

This report presents a novel data interpolation method in the phase-scrambling Fourier transform (PSFT) imaging technique[1]. A coarsely sampled PSFT signal which an aliasing artifact is produced in the reconstructed image can be interpolated to obtain fully encoded signal by using an iteration method based on the Gerchberg's algorithm[2] known as a super resolution technique. Modification of the Gerchberg's algorithm is made to apply the method to interpolation of PSFT signal. Numerical simulation using MR images shows that alias-free images are reconstructed from the interpolated PSFT signal by using this technique.

Data Interpolation in PSFT Imaging Technique by using Modified Gerchberg's Algorithm

The signal in the PSFT technique which uses a quadratic nonlinear field gradient pulse in syncronization with the phase encoding gradient pulse in the conventional Fourier imaging technique is given by Eq.(1).

$$v(\gamma g_x t_x, \gamma g_y t_y) = \int_{-\infty}^{\infty}\int \left\{\rho(x,y)e^{-j\gamma b\tau(x^2+y^2)}\right\}e^{-j(\gamma g_x t_x x+\gamma g_y t_y y)}dxdy \qquad (1)$$

where $\rho(x,y)$ is the spin density in the x-y plane at the slice selected z-location, γ is the magnetogyric ratio, b is the coefficient of the quadratic nonlinear field gradient, τ is its impressing time, g_x and g_y are the read out and phase encoding gradients, respectively. A coarsely sampled PSFT signal can be interpolated using modified Gerchberg's algorithm.

The modified Gerchberg's algorithm is described as follows.

i) Pad zero in the phase encoding direction (y-direction) between the coarsely sampled N x $N/2$ points PSFT data and make total data number N x N.

ii) Reconstruct image following Eq.1 by taking Fourier transform and multiplying by the quadratic phase term $e^{j\gamma b\tau(x^2+y^2)}$, and enlarge the image in size of $4N$ x $4N$. The image has an alias in the phase encoding direction (y-diection).

iii) Set the data in the imaginary part of the enlarged image equal to zero to destroy the phase coherency of the alias. (Gerenally, the spin density image is assumed to be a real number image.)

iv) Construct PFST signal in size of $4N$ x $4N$ from the image obtained in stage iii) by taking inverse Fourier transform and multiplying by the quadratic phase term $e^{-j\gamma b\tau(x^2+y^2)}$. Then replace the data of base-band region in size of N x N with original coarsely sampled data, and set the data outside of the base-band region both in the real and imaginary parts of the PSFT signal equal to zero.

v) Reconstruct image in size of $4N$ x $4N$ by taking Fourier transform and multiplying by the quadratic phase term $e^{j\gamma b\tau(x^2+y^2)}$.

Then stages iii) to v) are iterated many times. Finally, the base-band region in size of N x N is extracted from $4N$ x $4N$ PSFT data. The extracted N x N data are the interpolated PSFT signal from which alias-free image can be reconstructed.

Simulation

Numerical simulation was performed using MR images acquired in conventional MRI. A PSFT signal for phase scrambling parameter $\gamma b\tau = 0.614$ rad/cm^2, spatial resolution $\Delta x = \Delta y = 0.1$ cm at coarse sampling condition (data number $Nx = 256$, $Ny = 128$) was generated numerically by computer calculation and modified Gerchberg's algorithm was applied. Figure 1(a) shows a zero padded coarsely sampled PSFT signal and (b) is reconstructed image from (a). An alias is produced in the phase encoding direction. Figure 2(a) shows the interpolated PSFT signal and Fig.2(b) shows the reconstructed image. Almost of the alias is eliminated in the image.

Discussion

In the original Gerchberg's algorithm, two processes i) restriction of image extent using apriori knowredge on image extent in image space and ii) data replace with original band limited spectrum of the blurred image in spatial frequency space are iterated to extrapolate the band limited spectrum. The original Gerchberg's algorithm, however, cannot be used at all to interpolate a coarsely sampled PSFT signal. Two conditions that setting imaginary part of image equal to zero in stage iii) and setting outside of the base-band equal to zero in stage iv) are necessary to be converged the above described iteration procedure and to obtain interpolated PSFT signal.

This work demonstrates that it is feasible to obtain alias-free image from a coarsely sampled PSFT signal by data interpolation using modified Gerchberg's algorithm, and demonstrates a feasibility of new fast imaging method differed from the conventional parallel imaging. Advantage of this approach is that only a single signal receiving system is needed to impliment the alias-free image acquisition.

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References

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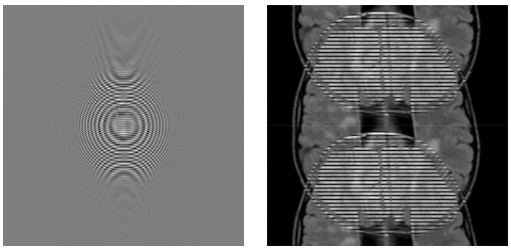
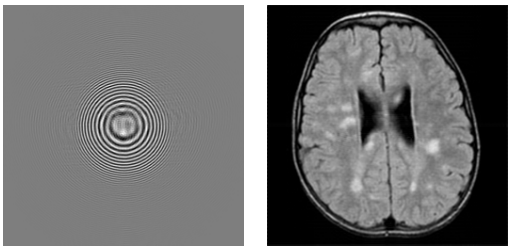


(a) **(b)**

Fig.1 (a): Zero padded coarsely sampled PSFT signal in row direction (phase encoding direction), (b): Reconstructed image from (a). Alias is produced in phase encoding direction.



(a) **(b)**

Fig. 2 (a): Interpolated PSFT signal after 40 times iteration, (b):Reconstructed image from (a). Aliasing artifact is not seen in the image.