# Combining Compressed Sensing and Parallel Imaging

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## Introduction

Compressed sensing and parallel imaging use fundamentally different acceleration methods. Compressed sensing relies on L1-norm minimization in a sparse transform space to allow reconstruction of randomly undersampled k-space data (1). Parallel imaging uses L2-norm error minimization to incorporate receive B1 information into the reconstruction of undersampled multicoil k-space data (2). An L1-norm penalty function has also been used in image denoising (3) and to denoise and regularize parallel imaging and non-Cartesian k-space reconstructions (4). It is desirable to combine these techniques for improved acceleration and robustness.

## Methods

In MRI the k-space data y are effectively the Fourier transform of a complex image m, i.e. y = Fm, where F is a 2D or 3D Fourier transform operator. For compressed sensing a sparsifying transform W is needed. Typically W is a wavelet or gradient transform (total variation). A central result of compressed sensing is that for sufficiently incoherent sampling (usually realized in MRI through random Fourier encoding), the image for undersampled single coil data can be reconstructed by minimizing $J(m) = \|Fm - y\|_2^2 + \lambda\|Wm\|_1$ where the constant $\lambda$ is adjusted to balance data fidelity and artifact reduction. Although only one sparsifying transform is shown here for simplicity, in practice two or more transforms are usually beneficial.

SENSE parallel imaging can be done by minimizing $J(m) = \|Em - y\|_2^2$ for undersampled multicoil data, where E is the coil encoding operator that includes both the B1 receive field and the Fourier operator F. Although a closed form solution for the minimum of J(m) exists for the parallel imaging case, when nonuniform or non-Cartesian k-space sampling is used, an iterative method such as the conjugate gradient (5) is usually used to find the minimum.

Compressed sensing and parallel imaging can be combined by minimizing $J(m) = \|Em - y\|_2^2 + \lambda\|Wm\|_1$ for randomly undersampled multicoil data. Setting $\lambda$=0 results in the multicoil parallel imaging solution, whereas replacing E with F results in the single coil compressed sensing solution. This solution can also be thought of as L1-norm regularized and denoised parallel imaging.

A 3T commercial scanner (GE Healthcare, Waukesha, WI) and 8-channel head coil (Invivo, Gainesville, FL) were used to scan a volunteer with a 2D T1-weighted spin echo protocol (axial plane, TE/TR=11/700, 22cm field of view, 10 slices, 256x256 matrix). Full Nyquist sampling was used for acquisition and 1D undersampling was simulated by discarding phase encoding lines. Gradient and wavelet transforms were used to sparsify the image. The gradient transform was implemented as a 2D nearest neighbor difference. The wavelet transform used Daubechies-4 wavelets. The $\lambda$ parameters were adjusted empirically.

A fully sampled sum of squares image was reconstructed for comparison to the accelerated cases. Three accelerated cases were simulated. (I) Single-channel compressed sensing was simulated by undersampling the Fourier transform of the sum of squares image. (II) Parallel imaging alone, and (III) compressed sensing plus parallel imaging were simulated by undersampling the acquired 8-channel k-space data. For each case, the solution was calculated by finding the minimum of the appropriate J(m) using a conjugate gradient algorithm with a maximum of 20 iterations. For all three cases, a Nyquist sampled region with radius equal to 10% of the fully sampled radius was included. For parallel imaging alone and compressed sensing plus parallel imaging, the Nyquist region was used to estimate coil sensitivities. For the case of parallel imaging alone, undersampling in the outer reduction area was uniform, whereas it was random for the cases involving compressed sensing. For all undersampled cases, 1D net accelerations as high as 3.3 were investigated.

## Results

The fully sampled sum of squares image and the three undersampled cases for acceleration factor 3.3 are shown in Figures 1-4. The result for compressed sensing alone (Fig. 2) has considerable blurring when the $\lambda$ parameters are adjusted to remove most of the undersampling artifacts. The result for parallel imaging alone (Fig. 3) has residual undersampling artifacts as well as increased noise, although the image is sharp. Using both compressed sensing and parallel imaging (Fig. 4) removes almost all of the undersampling artifacts and with minimal blurring. Although the acceleration here is relatively modest, using 2D undersampling is likely to allow higher compressibility and higher acceleration factors. This technique is relatively straightforward to implement because it can be combined with existing SENSE reconstruction methods.

## Conclusions

Compressed sensing, parallel imaging, denoising, and regularization can be combined. One method only requires replacing the Fourier operator in the compressed sensing formulation with the coil encoding operator appropriate for SENSE parallel imaging. Random undersampling appropriate to compressed sensing is still used to allow image recovery through L1-norm minimization. Coil sensitivities can be estimated using variable density Nyquist sampling of the center of k-space. The resulting acceleration and image quality are better than with any of the techniques alone.

## References

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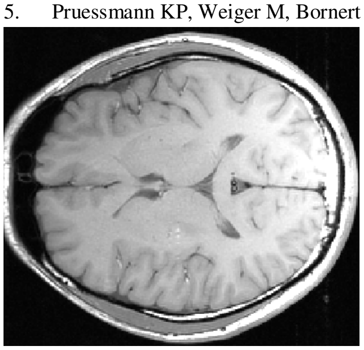

Figure 1. Fully sampled sum of squares image

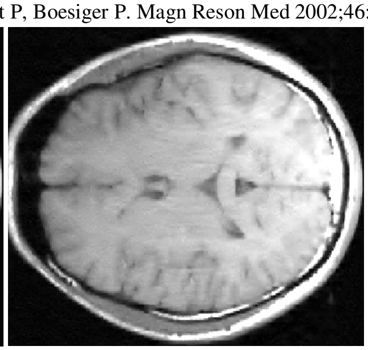

Figure 2. Compressed sensing alone

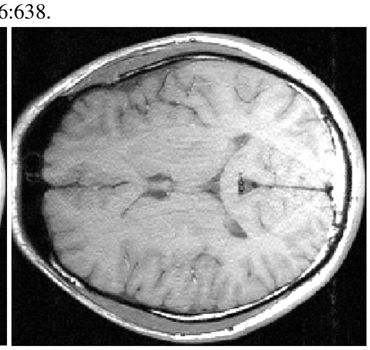

Figure 3. Parallel imaging alone

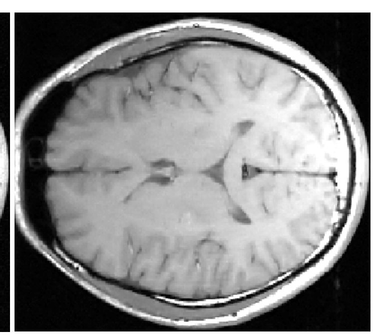

Figure 4. Compressed sensing plus parallel imaging