Introduction of a Nonconvex Compressed Sensing Algorithm for MR Imaging

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Introduction

The introduction of Compressed Sensing (CS) to the field of MRI offered a new perspective to speed up data acquisition [1,2]. Unfortunately, the acceleration in the experiment comes along with an increase in reconstruction time due to the need to determine of weighting parameters in the CS algorithm. The reconstruction algorithm suggested by Lustig et al. [1] performs a minimization of the sum of: 1) the squared l_2 norm (l_p norm: $\|x\|_p=(\sum_i |x_i|^p)^{1/p}$, $p\in \mathbf{R}$) of the differences between the reconstructed and acquired k-space data points; 2) the Total Variation (TV) norm [3]; 3) the l_1-norm of the wavelet coefficients. This method is further referred to as the "Convex method". The TV norm and Wavelet l_1 norm are each weighted by a parameter. If the Wavelet decomposition [2] were discarded, one parameter which must be determined would still remain. Recently, Chartrand [4] published an alternative to these proposals. His approach minimizes an l_p norm with $p < 1$, i.e. only one term remains; therefore, no parameters must be selected; this leads to a reduced computational load. This approach is referred as the "Nonconvex method". This work describes the first application of the nonconvex method to MR imaging.

Materials and methods

Both CS approaches were compared in a possible clinical application: Dynamic radial imaging. Radial imaging in combination with CS has been presented recently [2]. A radial cine dataset of a beating human heart was acquired with a 32-channel coil-array (Rapid Biomedical, Rimpar, Germany). The experiment was performed on a 1.5 T Avanto (Siemens Medical Solutions, Erlangen, Germany) clinical scanner. The dataset contained 21 timeframes, 192 readout points and 224 projections per timeframe, imaging parameters: 2D radial Turbo-FLASH, FOV 300x300 mm². A total of 32 projections was employed for the reconstructions. The data were processed in MATLAB (The MathWorks, Natick, USA). Accelerated acquisitions were mimicked by retrospectively removing radial projections. By subtracting the undersampled timeframe data from a temporally averaged composite dataset, sparse difference images can be created. The undersampled projections of each coil were gridded onto a Cartesian grid [5] followed by an adaptive coil combination procedure [6]. The CS reconstructed dynamic image was subtracted from the temporal average to obtain the final reconstruction. A version of the convex and nonconvex algorithms was implemented; for the convex approach, the specifications from [1] were followed; the nonconvex proposal was realized by minimizing $\|x\|_p$ with $p = 0.75$, where x is the complex-valued image. $\varepsilon = 1$ was diminished by a factor of 10 every 20th iteration, the algorithm terminated when $\varepsilon < 10^{-10}$.

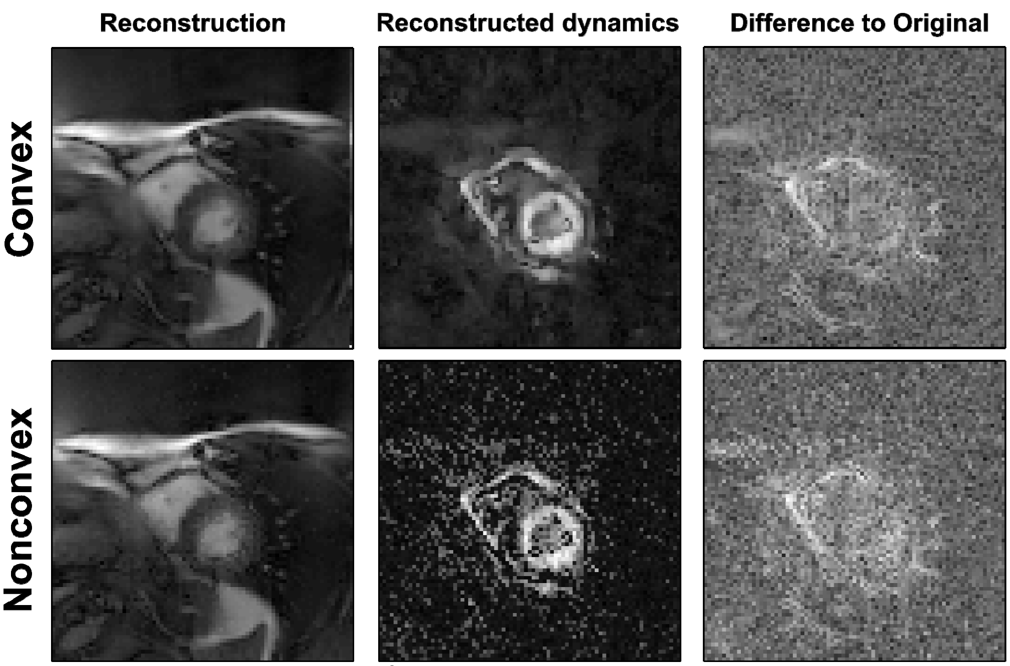

Figure 1: Reconstruction results. The left column can be obtained by subtracting the reconstructed dynamics (middle column) from the composite image of the dynamic dataset. The right column shows the differences (x 10) between the reconstruction from the fully encoded image

Results

Fig. 1 shows that both algorithms converge to similar results. The reconstructions utilized only 15 % of k-space (after gridding of 32 projections); both approaches offer a good agreement of the resulting images with the original timeframe image. While the convex algorithm required several hours to determine adequate parameter, the nonconvex approach converged within a few minutes.

Discussion

The convex approach has been shown to be a promising tool in accelerated MR imaging. It takes advantage of image denoising techniques; for this reason, the convex reconstruction appears smoother. Nonetheless, the parameter determination is a major disadvantage because requires a long computation time. Hence, the nonconvex algorithm is advantageous in this context. Even if both approaches performed the same number of iterations per unit time, the nonconvex method should outperform the convex in terms of computation time. Furthermore, Chartrand stated that the nonconvex approach should allow for higher acceleration factors than the convex method. However, this claim was not tested. Therefore, both methods will be subject to further investigation to determine their pros and cons.

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