Nonlinear Inversion with L1-Wavelet Regularization – Application to Autocalibrated Parallel Imaging

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Introduction:

To improve parallel imaging techniques with autocalibration, algorithms were recently presented which estimate the coil sensitivities and the image at the same time [1,2]. However, even with perfectly known coil sensitivites parallel imaging suffers from much lower SNR than to be expected from the reduced scan time alone. This is due to the reconstruction process (quantified by the g-factor map) and usually counteracted by regularization. Linear reconstruction methods are restricted to a regularization term which is related to the L2-norm of the image. Nevertheless, nonlinear regularization methods like the L1-norm in combination with a sparsity transform or total variation are known to suppress noise much more efficiently. This work demonstrates how L1-wavelet regularization can be incorporated into an autocalibrating parallel imaging algorithm based on a regularized nonlinear inversion method described earlier [2].

Methods:

The method is based on the idea that the L1-norm can be reduced to a L2-norm by a pointwise substitution $x \dashrightarrow z = x / \mathrm{sqrt}(|x|)$. In this way a reconstruction problem $G(x) = y$ with L2-regularization $|x|_2$ is transformed to a non-linear problem $F(x) = G(|z|z)$ where the L2-regularization in the substituted variable z corresponds to the desired L1-regularization for the original variable x: $|z|_2 = |x / \mathrm{sqrt}(|x|)|_2 = |x|_1$. To achieve noise suppression the system has to be transformed with a sparsity transform in the image domain before applying the substitution. A transform based on the *Cohen-Daubechies-Feauveau 9/7* wavelet was used for this purpose. The resulting non-linear system is solved with a regularized inversion algorithm based on a Newton-type method: Given an estimate x_n the next estimate $x_{n+1} = x_n + dx$ is calculated by solving the linearized equation $DF(x_n)dx + F(x_n) = y$ for the update dx. For each Newton step this linear system is solved by the conjugate gradient algorithm in combination with a suitable L2-regularization term. The specific algorithm used is known as Iteratively Regularized Gauss Newton Method (IRGNM). For a linear operator G the resulting algorithm is related to the Iteratively Reweighted Least Squares (IRLS) algorithm. In this work G is the nonlinear operator which maps the coil sensitivities and the image vector to the predicted data in k-space. Regularized inversion then yields the coil sensitivities and the unfolded object at the same time. The coil part is regularized with a Sobolev-norm preserving smoothness as presented earlier [2], whereas the object part is regularized with a L1-wavelet norm as described above.

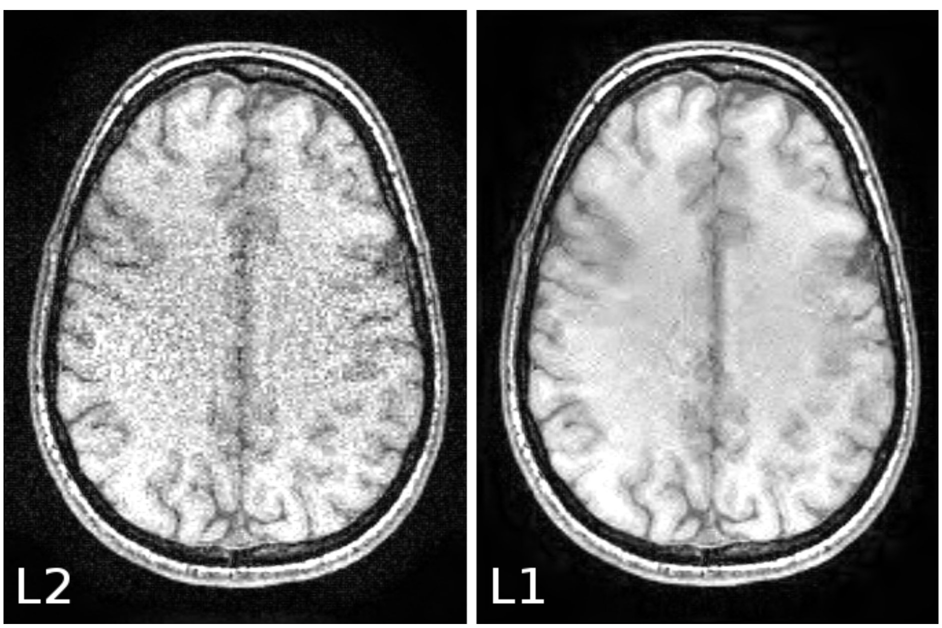

Figure: Images from a human brain acquired with parallel imaging and reduction factor 6 = 3×2 and reconstructed with a Newton method.

(**Left**) Reconstruction with L2-regularization.

(**Right**) Reconstruction of the same data with a L1-term applied in a wavelet basis.

Results und Discussion:

As an example the proposed method was applied to human brain images acquired with parallel imaging (3D FLASH; TR/TE = 10.6/4.2 ms, flip angle 17°; image matrix 256×162×224) at a reduction factor of 6 = 3×2 (12 receiver channels; 24×24 reference lines). Images were reconstructed both with a L1-term applied in the wavelet basis and with a conventional L2-term. In both cases the iteration was stopped as soon as no undersampling artefacts were visible anymore. As expected, the reconstruction with L1-wavelet regularization exhibits much less noise than the L2-regularized image. L1-regularization with a Newton-type algorithm presents itself as a simple alternative to other non-linear optimization methods. An interesting extension would be the application to irregular sampling patterns to achieve further acceleration as promised by the compressed sensing theory.

References: **1.** Ying L, Sheng J. Magn Reson Med 2007;57:1196-1202 **2.** Uecker et al., Proc Int Soc Magn Reson Med 2007; 15:1740