k-Space Sampling for Motion Correction with Parallel Imaging Techniques

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Introduction: Motion related artifacts are a common cause of image corruption in MRI. Sources of artifact include discrete and infrequent involuntary motions of sedated or uncooperative patients as well as the more familiar respiratory and cardiac motions. In all cases motion artifacts can severely degrade the diagnostic quality of MRI examinations. Redundant information in multiple receiver coil data may facilitate correction of the images if the motion is measured. A previous treatment of the general motion correction problem (1) solved the matrix inverse for multiple deformations applied in image space. Here, we formulate the problem in *k*-space: motion of the object is interpreted as a modification of the phase encoding functions which map the object into *k*-space. If the motions are in some way known, then all the information required to formulate the generalized encoding matrix is known and, in principle, an image may be reconstructed by an appropriate inverse. General motions and deformations are tractable as it is possible for irregularly distributed *k*-space data to be reconstructed using the formulation of the inverse of the generalized encoding matrix, given knowledge of the sampling pattern. Conceptually in *k*-space, motion correction is possible given the effective remapping of *k*-space from the known motion parameters and, analogously to parallel imaging techniques, redundant information acquired from receiver coil arrays is used to reconstruct areas of *k*-space left sparsely sampled after motion corrupts signal encoding. In this way, uncorrupted images of moving objects may be reconstructed. Addressing the scenario of discrete head motions during an acquisition, we will investigate possible trajectories in *k*-space that will optimize the use of the redundant information available from the array coil for use in the motion correction problem. We demonstrate the *k*-space formulation for a simulated 2-D image acquisition disrupted by in-plane rigid-body translation and rotation as may commonly occur in prolonged examinations of the head in sedated or uncooperative patients.

Theory: The standard formulation for parallel imaging reconstruction (2) describes the acquired signals in terms of the imaged object by the operation of a generalized encoding matrix (GEM) (3), with image reconstruction being achieved by a suitable inversion of the encoding matrix. Conceptually, the encoding matrix comprises multiplication by the sensitivity profiles of the individual elements of the coil array and Fourier Transformation into *k*-space. Here, we propose that by including motion and other deformations of the object into the encoding matrix the GEM formulation offers a framework for reconstructing an uncorrupted image from an object suffering arbitrary motions. We treat the image reconstruction problem separately from the measurement of motion, assuming that the motion parameters are known, which may be extracted from navigator techniques or mechanical devices, and that the coil sensitivities are known, which may be measured in vivo with rapid low-resolution images (4).

Defining the object $\tilde{\rho}(\mathbf{r^0})$ in its own 'body frame' of position coordinates $\mathbf{r^0}$ at time t=0, positions in the arbitrarily deformed object are described by 'scanner' coordinates $\mathbf{r}(t)=\mathbf{F}(\mathbf{r^0},t)$ where the deformation is described by the transformation of coordinates $\mathbf{F}$ so that $\tilde{\rho}(\mathbf{r^0})=\rho(\mathbf{r}(t),t)$. The GEM formulation for the signals $S(\alpha,l)$ acquired at time t (indexed by α) for k-space positions $\mathbf{k}(\alpha)$, and in coil element l, with coil sensitivity and encoding functions written in the 'scanner frame,' may be transformed into 'body frame' coordinates $\mathbf{r^0}$ by the known deformation, where the Jacobian of the transformation $|\partial\, \mathbf{F}(\mathbf{r^0},t) / \partial\, \mathbf{r^0}|$ is included [Eq. 1]. The uncorrupted original image at time t=0 in the 'body frame' $\tilde{\rho}(\mathbf{r^0})$ may be recovered by solving the inversion problem for the motion-adjusted generalized encoding matrix $\mathbf{B}^{\mathrm{ma}}(\mathbf{k}(\alpha),\mathbf{r^0})$ identified by the curly braces in Eq. 1.

Eq.1

$$S_{\alpha,l} = \int C_l(\mathbf{r})\cdot\exp(i\,\mathbf{k}_\alpha \mathbf{r})\cdot\rho(\mathbf{r}(t),t)\,d\mathbf{r} = \int\left\{C_l\left(\mathbf{F}(\mathbf{r^0},t)\right)\cdot\exp\left(i\,\mathbf{k}_\alpha\cdot\mathbf{F}(\mathbf{r^0},t)\right)\cdot\left|\frac{\partial\mathbf{F}(\mathbf{r^0},t)}{\partial\mathbf{r^0}}\right|\right\}\tilde{\rho}(\mathbf{r^0})\,d\mathbf{r^0}$$

Methods: We implemented and tested this technique using simulations of a 2-D image acquisition corrupted by in-plane rigid body translation and rotation. Motion-corrupted signals were generated for the spin density which was moved at three discrete time-points during signal acquisition as shown in Fig. 1. Coil sensitivity profiles were computed for a typical 8-channel head array coil by a quasi-static Biot-Savart method. A standard fully sampled Cartesian trajectory of *k*-space was assumed. Interleaving of acquired phase encoded readouts, in the manner of two accelerated acquisitions (offset by one phase encode position), was assumed. This trajectory was intended to cover *k*-space in the original 'body frame' as uniformly as possible. Given rotational motions during the acquisition the interleaving reduces the extent of undersampling introduced, when compared to linear traversal, by the discrete motions occurring between segments of the acquisition as demonstrated in Fig. 2. It was assumed that motion is effectively frozen during frequency-encoded readouts. The image matrix was 64×64 and interpolated to 512×512. The GEM inversion problem was solved iteratively by a conjugate gradient method (5).

Results and Discussion: The proposed method for motion correction shows promising results, eliminating the bulk of artifacts as seen in Fig. 3 for images reconstructed from motion corrupted signals with conventional GEM reconstruction and with the motion-adjusted GEM formulation. Image artifacts are reduced by effectively over-sampling with parallel imaging in order to reduce the extent to which *k*-space deformations resulting from motion leave regions sampled below the Nyquist criterion. Interleaved phase encode acquisition, nearly analogous to serial parallel accelerated image encoding, is preferable if the time spectrum of motion is dominated by low frequencies. The motion-adjusted GEM formulation is in principle applicable to arbitrary deformations which may be particularly useful in correcting respiratory motions in abdominal imaging applications for patients who are unable to control breathing effectively. The principal requirement for this motion correction method is that the motion parameters are known before image reconstruction; a previous study used iterative methods to find solutions for unknown motions (1). In general, the inversion of the entire encoding matrix is a lengthy procedure. Regularization techniques are under investigation for minimizing required computation time. In addition, the motion-adjusted GEM formulation may incorporate parallel imaging reconstruction for accelerated acquisition of undersampled signal data.

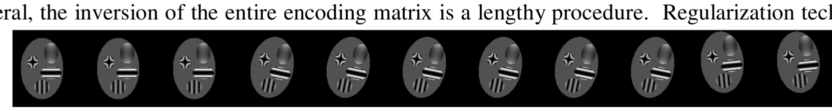

Figure 1: 'Snapshots' of the simulated object as a function of time (left-to-right) showing three discrete changes in position by rigid-body translation and rotation.

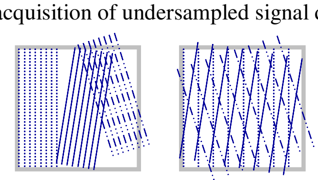

Figure 2: Sampled *k*-space (blue lines) showing different undersampling after three discrete rotations of the object, with a linear (left), and interleaved "parallel imaging-like" traversal of *k*-space (right).

Conclusion:
The motion-adjusted GEM formulation demonstrated here shows successful reduction of motion related artifacts. The method is in principle not specific to any particular traversal of *k*-space but different *k*-space acquisition orders will affect the quality of the correction. The method is a promising technique for future development.

References: 1. Batchelor, MRM 2005;54:1273. 2. Pruessmann, MRM 1999;42:952. 3. Sodickson, MedPhys 2001;28:1629. 4. McKenzie, MRM 2002;47:529. 5. Pruessmann, MRM 2001;46:638.

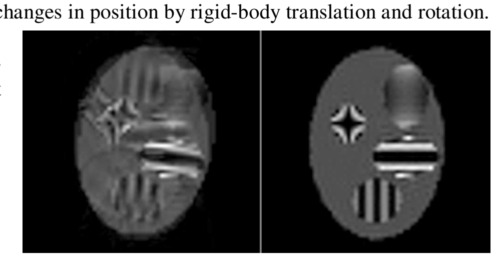

Figure 3: Images of the object reconstructed from motion-corrupted signals: with conventional GEM (left) and motion-corrected with the motion-adjusted GEM formulation (right).