Spiral Projection Imaging Motion Correction Using Lines of Intersection

K. O. Johnson[1], and J. G. Pipe[1]
[1]Barrow Neurological Institute, Phoenix, AZ, United States

Introduction In Spiral Projection Imaging (SPI), planes of data are collected to fill a sphere in k-space [1]. Planar orientations have been proposed that would lead to redundant data collection in order to estimate motion in all six degrees of freedom [2]. Every plane intersects every other plane on a line. Therefore, any two planes can be compared, to determine which line from each plane is common between them. Planar orientations can be deduced that are consistent with the lines of intersection.

Methods The orientation of any plane *i*, can be defined by a rotation matrix . The normal of this plane, n_i , corresponds to the rotation of a base normal n_0 , or

$$n_i = \mathrm{R}_i n_0 \quad (1)$$

Given two planes *i* and *j* with normals n_i and n_j, the intersection of these two planes would fall on a line represented by the 3-D vector $a_{i,j}$, where

$$a_{i,j} = n_i \times n_j \quad (2)$$

In the frame of reference to plane *i*, this line resides in plane and would be represented by the 2-D vector

$$\hat{a}_{i,j} = \mathrm{R}_i^{-1} a_{i,j} \quad (3)$$

The phase of each $\hat{a}_{i,j}$ can be estimated from the scanned data, by comparing the magnitude data from every pair of planes. Each chord that passes through the origin from one plane is compared to each chord of another plane. The chords that are the best match are chosen, since their data should be equivalent. These equations demonstrate the relationship of all of the planar orientations with values that can be calculated. This relationship is a non-linear system of equations which can be solved to provide a solution of planar orientations consistent with the estimated lines of intersection. The system is over determined when there are at least three different planes.

In a similar way a linear system of equations can be constructed to calculate the translation incurred in image space for each plane. The image space translational motion of every plane, t_i , is apparent with respect to the lines of intersection by

$$m_{i,j} = (t_i - t_j) \cdot a_{i,j} \quad (4)$$

Where $m_{i,j}$ is the image space translation motion that is manifested inline. Note that $m_{i,j} = m_{j,i}$, and therefore only one estimate is created for every pair. This system is also over determined when more than six planes are compared.

Results An unoptimized program to estimate the in plane intersections and solve for the planar orientations was implemented. Data for 123 planes (190 matrix) were synthesized with modest noise (rotational motion only). Newton's method was the chosen nonlinear solver. The computation time, to apply the rotation correction, was 5 minutes on a 3.0GHz processor. The error of estimates of $\hat{a}_{i,j}$ averaged 0.26 degrees off (8.6 degrees maximum). The error of orientation parameters averaged 0.24 degrees (4.1 maximum).

Discussion This algorithm presents an efficient approach to 3-D motion correction. Due to the nature of this algorithm, the solution is independent of the severity of motion. This will ensure consistent performance and consistent computational time. Further, the accuracy of final estimate is not dependent on the number of planes collected but the accuracy of the estimates of each $\hat{a}_{i,j}$. This accommodates under-sampling the number of planes.

There are several challenges to this approach. The most significant challenge to this approach is the application of this algorithm to multicoil data.

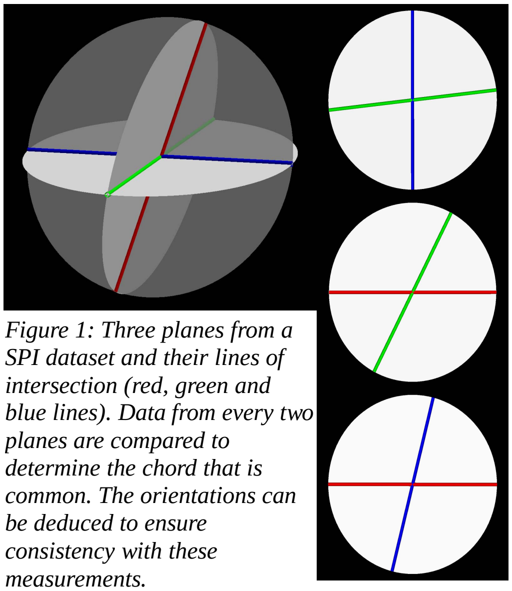

Figure 1: Three planes from a SPI dataset and their lines of intersection (red, green and blue lines). Data from every two planes are compared to determine the chord that is common. The orientations can be deduced to ensure consistency with these measurements.

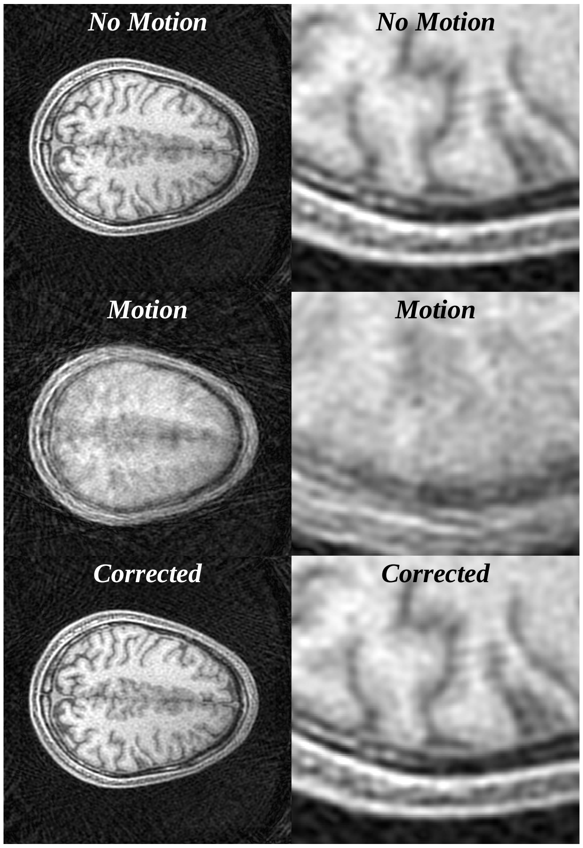

Figure 2: Slice from simulated data with close-up. A slowly varying non-coherent phase was added to eliminate symmetry of purely real data. Multi-coil data was not simulated.

References [1] Irrazabal P, Nishimura DG, *Mag. Res. Med.* 1995. 33; 656-662. [2] Robison RK, Pipe JG. *Proc ISMRM 2007*. # 1664.