Estimation of the Inversion-Pulse Efficiency in the Context of Multi-Exponential Analysis of Spin-Lattice Relaxation

C. Labadie^{1,2}, J-H. Lee³, and H. E. Möller¹

¹Max Planck Institute for Human Cognitive and Brain Science, Leipzig, Germany, ²Faculty of Physics and Earth Science, University of Leipzig, Germany, ³Biomedical Engineering, University of Cincinnati, Cincinnati, Ohio, United States

Introduction: Spin-lattice relaxation in the human brain is widely considered mono-exponential both in the white and gray matter. To account for imperfect inversion pulses, T_1 maps are typically computed by fitting a set of linearly sampled Look-Locker images following an inversion pulse with a nonlinear least-squares (NLLS) procedure including a parameter to estimate the inversion pulse efficiency [1]. We recently observed at 4 Tesla an additional small peak at a short T_1 (ca. 200 ms) in the relaxogram of a human volunteer by analyzing geometrically sampled inversion-recovery data by inverse Laplace transform [2] with the package CONTIN [3]. The purpose of this study is to design a procedure to estimate the efficiency of the inversion-pulse in the context of a multi-exponential spin-lattice relaxation analysis.

Simulations: A three-parameter mono-exponential NLLS analysis [1] may mistakenly fit a small peak at a short T_1 by underestimating the inversion-pulse efficiency as shown in the simulation of Fig. 1. T_1 relaxograms, $s(T_1)$, were simulated with a main peak at 1 sec and an additional peak with varying relative intensity (from 0 to 40%) and T_1 (from 100 ms to 1 sec); other parameters included 64 logarithmically sampled grids between 80 ms and 3.16 sec, Gaussian peakshape (width 0.05 in log10), inversion-pulse efficiency of 95%. The magnetization of the inversion recovery, M(TI), was computed without noise at 64 TI geometrically sampled between 24.4 ms and 11.43 sec: $M(TI)=\sum s(T_1) \times (1 - (2 \times 0.95) \exp(-TI/T_1))$. The three-parameters NLLS fit (Matlab *lsqcurvefit*) was performed without any constraint: $M(TI) = M_0 \times (1 - (2 \times b/100) \exp(-TI/T_1))$. When the relative intensity of the small peak is greater than 10% and its T_1 position smaller than 400 ms, the estimated efficiency of the inversion pulse, *b*, is significantly underestimated by the NLLS fit despite excellent sampling of the TIs.

To our knowledge, the CONTIN package does not directly permit to define a user kernel including a fitting parameter to account for the inversion-pulse efficiency. A procedure to estimate the efficiency was elaborated by using a grid search and choosing the value leading to the best chosen CONTIN solution according to the PROB1(α) criterion, a Fisher's *F*-distribution comparing the unregularized reference solution with that obtained by applying the regularization factor α . As in CONTIN, we selected the efficiency yielding the solution with the PROB1(α) nearest to 0.5 in our procedure. Solutions with correlated residuals were excluded according to the probabilities of uncorrelation for 1 to 3 adjacent residuals (PUNCOR), as well as those in which relaxogram edges were taking up more than 10% of the total signal (artefacts corresponding to under- or overestimated efficiencies). Fig. 2 shows the comparison of the NLLS fit with the CONTIN-based grid search. The simulated efficiency was kept constant to 95% (dashed line), and the relative intensity of the smaller peak to 20%, its T_1 position, however, varied between 100 ms and 1 sec. Normally distributed noise was added to yield an SNR of 100 (Matlab *normrnd*) and repeated 28 times. Unlike the three-parameter fit, the CONTIN-based grid search does not underestimate the inversion-pulse efficiency varied between 82% and 100% (dashed line). The CONTIN-based search slightly overestimates the efficiency by 1.56 ± 3.03 % whereas the three-parameter fit underestimates it by -5.40 ± 1.25%.



Fig. 4

In-vivo results: Two sets of inversion-recovery images (PURR) were measured at 4 Tesla with 64 geometrically spaced *TI* values [4]. The resolution of the 192×256 images was downscaled by summing 4 pixels and yielded images with SNR of 27.1 (volunteer A in Fig. 4) and 31.6 (volunteer B in Fig. 4). The efficiency maps for the CONTIN-based search were smoothed with an 8-mm FWHM Gaussian filter. Unlike the CONTIN-based search, the maps computed with three-parameter fit display lower estimated inversion pulse efficiencies in white matter, which erroneously leads to a distinct anatomical pattern. These results suggest that a small fraction of water is relaxing with a T_1 smaller than 400 ms in the white matter consistent with recent results [2].

Conclusion: It is possible to estimate the efficiency of an inversion pulse in the context of multi-exponential spin-lattice relaxation by performing a grid search based on the CONTIN criterion used to choose a regularized solution. Such estimation is a great asset to investigate the impact of water compartmentalization on T_1 relaxation. Our results suggest that the small T_1 peak, observed at ca. 200 ms in a separate data-set [2], was not due to the B_1 inhomogeneity expected at higher B_0 field but to a multi-exponential T_1 relaxation. The CONTIN-based procedure has a higher variance than the three-parameter NLLS fit. Further improvements may be achieved by increasing the constrain on the relaxogram edges (e.g., regularization orders higher than 2) and better defining the criteria to exclude solutions with correlated residuals.

References: [1] Ogg R.J. & Kingsley P.B., *Magn. Reson. Med.* 51 (2004) 625. [2] Labadie C. et al. *Proc. Intl. Soc. Mag. Reson. Med.* 15 (2007) 2106. [3] Provencher S.W., *Comput. Phys. Commun.* 27 (1982) 229. [4] Lee J-H., *Magn. Reson. Med.* 43 (2000) 773.

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