Interpolated Estimates of Magnitude, R2*, and Frequency Offset in SS-PARSE Imaging

W. Tang[1], S. J. Reeves[1], and D. B. Twieg[2]

[1]Electrical & Computer Engineering, Auburn University, Auburn, AL, United States, [2]Biomedical Engineering, University of Alabama at Birmingham, Birmingham, AL, United States

Introduction

Traditional MRI methods implicitly assume that local magnitude and frequency do not change during signal acquisition. Single-shot parameter assessment by retrieval from signal encoding (SS-PARSE) proposed in [1] relaxes this assumption. Because each datum is modeled as a sample from *(k, t)*-space rather than *k*-space, reconstruction based on this more accurate model cannot be directly implemented by FFT. Conjugate-gradients (CG) is used to simultaneously to estimate local image magnitude, decay, and frequency. Limited data from short read-out time makes reconstruction at high resolution an underdetermined inverse problem. However, a high-resolution model is desirable for modeling large decay and frequency gradients due, for example, to susceptibility effects and tissue boundaries. This paper proposes cubic interpolation to address this problem. To estimate coefficients with a lower resolution and use interpolation instead of using higher resolution directly keeps reconstruction determined or overdetermined. Because the gradients computation is one of the key parts in the CG algorithm, an efficient method is proposed to compute gradients with respect to interpolation coefficients. The computational complexity of this CG algorithm based on interpolation is determined by the resolution of the reconstructed images rather than the resolution of the coefficients. By using a polynomial approximation of the exponential time function of decay and frequency evolution [2], evaluation of cost and gradients in the CG algorithm can be accomplished with a linear combinations of FFTs. A line search strategy for CG is also addressed.

Method

The discrete version of the SS-PARSE model is written as $s(n) = \sum_i M_{0i} e^{nW_i} e^{-j\mathbf{k}_n \cdot \mathbf{x}_i}$, where $\mathbf{M}_0$ is the local transverse magnetization immediately following excitation, $\mathbf{W} = -(\boldsymbol{R}_2^* + j\boldsymbol{\omega})\Delta t$, $\boldsymbol{R}_2^*$ is the local net relaxation rate and $\boldsymbol{\omega}$ is the local frequency, Δt is the sampling interval, $\mathbf{k}_n$ is the trajectory and $\mathbf{x}$ is the location. Our goal is to reconstruct $\mathbf{M}_0$ and $\mathbf{W}$ from the observed signal $\mathbf{s}$. An iterative CG algorithm is used to minimize $J(\mathbf{z}) = \|\mathbf{s} - \hat{\mathbf{s}}(\mathbf{z})\|^2$ with respect to $\mathbf{z} = \{\mathbf{M}_0, \mathbf{W}\}$. We use cubic convolution interpolation [2] to represent $\mathbf{z}$, where the 1-D interpolation function is $g(x) = \sum_k c_k \, u[(x - x_k)/h]$. In this function, h represents the sampling increment, the x_k's are the interpolation nodes, u is the interpolation kernel, and the c_k's are coefficients which depend upon the sampled data. The kernel $u(x)$ used in this paper is defined as the left equation. By this interpolation, $\mathbf{z}$ is constructed from $\mathbf{A}^T\mathbf{C}\mathbf{A}$, where $\mathbf{A}$ is a sparse matrix made by $u(x)$ and $\mathbf{C}$ is the coefficient matrix to be estimated. It is difficult to directly compute $\nabla J/\nabla \mathbf{C}$. Let $\mathbf{u} = [u(-2+h) \quad u(-2+2h) \quad \cdots \quad u(2-h)]$. Then $\mathbf{B} = \mathbf{u}^T\mathbf{u}$. We can show that $\nabla J/\nabla \mathbf{C} = \mathbf{B} * \nabla J/\nabla \mathbf{z}$. Since $\mathbf{B}$ is a small matrix, computation of this convolution is fast.

$$u(x) = \begin{cases} \frac{3}{2}|x|^3 - \frac{5}{2}|x|^2 + 1 & 0 < |x| < 1 \\ -\frac{1}{2}|x|^3 + \frac{5}{2}|x|^2 - 4|x| + 1 & 1 < |x| < 2 \\ 0 & 2 < |x| \end{cases}$$

With interpolation, we are working on this inverse problem at a higher resolution. The exponential time function prevents us from using FFTs to accelerate the CG algorithm. This problem can be solved by polynomial approximation. A similar method, time- and frequency-segmentation [4], requires intensive computations for updating coefficients of approximation for the changing $\mathbf{W}$ in each iteration of the CG algorithm. Updating of coefficients of the polynomial approximation with the updating of $\mathbf{W}$ is unnecessary. We approximate e^{nW_i} as $\sum_{l=0}^{L-1} a_l n^l W_i^l$. The required order of this approximation is determined by $\boldsymbol{\omega}$ since experimental evidence shows that $\boldsymbol{R}_2^*$ is typically much smaller than $\boldsymbol{\omega}$. So polynomial approximations only need to cover half of the range of local frequency. Let $\omega_0 = (\omega_{min} + \omega_{max})/2$, $\mathbf{Z}^l = a_l(\mathbf{W} - j\omega_0)$ for $\mathbf{Im}\{\mathbf{W} - j\omega_0\} \geq 0$ and $\mathbf{Z}^l = a_l^*(\mathbf{W} - j\omega_0)$ for $\mathbf{Im}\{\mathbf{W} - j\omega_0\} < 0$. Then we have $e^{n\mathbf{W}} \approx e^{jn\omega_0} \sum_{l=0}^{L-1} n^l \mathbf{Z}^l$. By this approximation, we obtain the following approximations (box at left).

$$s(n) \approx e^{jn\omega_0} \sum_{l=0}^{L-1} n^l \sum_i M_{0i} Z_i^L \, e^{-j\mathbf{k}_n \cdot \mathbf{x}_i}$$

$$\frac{\partial J}{\partial M_{0i}} \approx \sum_{l=0}^{L-1} Z_i^l \sum_n [\hat{s}(n) - s(n)] n^l e^{jn\omega_0} e^{-j\mathbf{k}_n \cdot \mathbf{x}_i}$$

$$\frac{\partial J}{\partial W_i} \approx M_{0i} \sum_{l=1}^{L} Z_i^l \sum_n [\hat{s}(n) - s(n)] n^l e^{jn\omega_0} e^{-j\mathbf{k}_n \cdot \mathbf{x}_i}$$

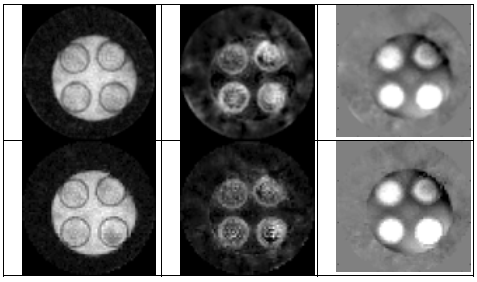
These three equations can be evaluated by FFTs. Our experiments show that it is unnecessary to use non-uniform FFTs to compute these FFTs. One can simply discretize $\mathbf{k}_n$ to an equally spaced grid whose resolution is higher than the resolution of the reconstructed images, to evaluate the equations with FFTs. For example, if the resolution of the reconstructed images is 64 by 64, $\mathbf{k}_n$ can be discretized to a 128 by 128 grid.

Line search of the CG algorithm is critical for a non-quadratic problem. For this problem, the line search is to minimize $J(\mathbf{C} + \alpha\mathbf{C}')$ with respect to α. Using golden section search to find minimizer $\alpha_{\min}$ requires many evaluations of $J(\mathbf{C})$. For this inverse problem, $J(\mathbf{C} + \alpha\mathbf{C}')$ is close to a quadratic function of α. Since the value of $J(\alpha = 0)$ is the value of $J(\mathbf{C})$ from the results of the previous iteration, we only need to evaluate $J(\alpha = \pm\Delta)$ or $J(\alpha = \Delta)$ and $J(\alpha = 2\Delta)$ to formulate a quadratic function of α to find $\alpha_{\min}$, where Δ is a scalar. Our experiments show that using $J(\alpha = \pm\Delta)$ gives better results. Except the initialization, each iteration of the CG algorithm used here is summarized in the right box.

1. Use quadratic approximation to find $\alpha_{\min}$ to minimize $J(\mathbf{C} + \alpha\mathbf{C}')$.
2. $\mathbf{C} = \mathbf{C} + \alpha_{\min}\mathbf{C}'$, $\mathbf{z} = \mathbf{A}^T\mathbf{C}\mathbf{A}$, evaluate $J(\mathbf{z})$.
3. Compute $\nabla J/\nabla \mathbf{z}$, $\nabla J/\nabla \mathbf{C} = \mathbf{B} * \nabla J/\nabla \mathbf{z}$.
4. Use Polak–Ribière method to update $\mathbf{C}'$ [5].

Results

The methods described in the previous section are applied to a phantom experiment using a 4.7T Varian MRI system. This phantom is a cylinder containing water and four tubes with agarose gel. Images from two different methods are compared. Images of the first row, 128 by 128 resolution, are reconstructed by cubic convolution interpolation with polynomial approximation. Images of the second row, 64 by 64 resolution, are directly reconstructed (no interpolation), and polynomial approximation is not used. From left to right, the images are local magnitude, decay, and frequency. The interpolation method not only gives sharper edges of the circles but also gives smoother insides of the circles, particularly for the image of local decay. We also compared the two methods for noiseless synthesized data, we saw more obvious improvements.



With polynomial approximations of the exponential time function, the CG algorithm is accelerated. It takes 166 seconds to reconstruct the 64 by 64 images in the second row, while it only takes 46 seconds to reconstruct the 128 by 128 images in the first row. Without the fast algorithm, it takes 877 seconds for the first row. We applied quadratic line search to both "fast" and "slow" methods. Without quadratic line search, the run times are doubled. All these computations are performed in MATLAB on a computer made by @Xi Computer Corporation, which is equipped with 2 AMD Opteron 2.4GHz dual-core CPUs and 16GB RAM. Only one core was used in the computation.

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References

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