Water-Fat Decomposition with Regularized field map

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INTRODUCTION: Water-fat decomposition methods based on chemical shift, such as IDEAL and the multipoint Dixon approach [1,2], have a variety of clinical uses. However, these methods can be sensitive to field inhomogeneity. Typically these methods rely on serial reconstruction approach with separate field map estimation with subsequent smoothing in an attempt to reduce noise, and final fat-water separation. In this work, we describe a penalized-likelihood (PL) method for jointly estimating water, fat, and the field map. The method uses the a priori knowledge that field maps are usually smooth by including a regularization term in the cost function. The fat and water components can be solved for analytically as a function of the (unknown) field map, simplifying the optimization problem to be a function of the field map only. The proposed method includes an iterative algorithm that monotonically decreases that cost function. By providing a field map estimate that is inherently smooth even in regions of low signal intensity, the method may yield improved water and fat images.

METHODS: The input data is L reconstructed images with different echo times, denoted by $\underline{y} = (y^1, y^2, \ldots, y^L)$, where typically $L=3$. Let Δf denote chemical shift of fat relative to water (Hz) and ω_j denote the off-resonance of the jth voxel. We model the L images by $y^l_j = (W_j + F_j e^{i\,2\pi\Delta f\,t_l})\, e^{i\,\omega_j t_l} + \varepsilon^l_j$ for $l = 1,2,.. L$, where t_l denotes the echo time difference of the lth scan relative to the original scan, W_j and F_j denote complex water and fat components in the jth voxel, respectively, and ε^l_j denotes the complex noise. The goal here is to jointly estimate the field map $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_N)$ and the water and fat components $\boldsymbol{W} = (W_1, W_2, \ldots, W_N)$ and $\boldsymbol{F} = (F_1, F_2, \ldots, F_N)$ from the images $\underline{y}$. We assume that the additive complex noise values, ε^l_j, are independent Gaussian random variables with the same mean equal to 0 and variance σ^2. Finding the ML estimates of **F**, **W**, and $\boldsymbol{\omega}$ as [1] ignores the important prior knowledge that field maps are usually spatially smooth functions. Instead of low pass filtering, we jointly estimate ***W, F**, and* $\boldsymbol{\omega}$, by including a spatial roughness penalty $R(\omega)$ in the PL cost function:

$$(\hat{\boldsymbol{\omega}}, \hat{\mathbf{W}}, \hat{\mathbf{F}}) = \underset{\boldsymbol{\omega}\in\mathbb{R}^N\, \mathbf{W},\mathbf{F}\in\mathbb{C}^N}{\arg\min}\ \Psi_{\mathrm{PL}}(\boldsymbol{\omega}, \mathbf{W}, \mathbf{F}),\ \ \Psi_{\mathrm{PL}}(\boldsymbol{\omega}, \mathbf{W}, \mathbf{F}) = \sum_{j=1}^{N} \left\| \underline{y}_j - D(\omega_j) A \mathrm{X}_j \right\|^2 - \beta R(\boldsymbol{\omega}) \quad , \quad (1)$$

where $D(\omega_j) = diag(e^{i\omega_j \underline{t}})$, $A = [\underline{1} \quad e^{i\Delta f \underline{t}}]$, $X_j = [\, W_j \quad F_j\,]$, and $\underline{y}_j = (y^1{}_j\, y^2{}_j \ldots y^L{}_j)$. When ω is an $N_1 \times N_2$ field map $\omega[n,m]$, the regularizing roughness penalty uses differences between horizontal and vertical neighboring voxel values as follows:

$$R(\boldsymbol{\omega}) = \sum_{n=1}^{N_1-1} \sum_{m=0}^{N_2-1} \psi(\omega[n,m] - \omega[n-1,m]) + \sum_{n=0}^{N_1-1} \sum_{m=1}^{N_2-1} \psi(\omega[n,m] - \omega[n,m-1]) \quad , \quad (2)$$

where for simplicity $\psi(t) = t^2/2$. To minimize (1), we first estimate $\boldsymbol{W}$ and $\boldsymbol{F}$ as a function of $\boldsymbol{\omega}$ and substitute into (1) yielding a function of $\boldsymbol{\omega}$ only. We then minimize over $\boldsymbol{\omega}$ using an optimization transfer iteration that decreases the cost function monotonically. Thus the estimated field map converges to a local minimizer of the PL cost function. To simplify selecting the regularization parameter β in (1), we normalize the image magnitudes by the median of the values of the first data set so that the "typical" value is unity. To encourage the iteration to converge to a desirable local minimum, we choose the initial field map $\boldsymbol{\omega}^{(0)}$ for which the ML cost function is the smallest among all its elements are in the set $S = \{|\Delta f/2|\cdot k/n : n = 50, k = -n, \ldots, n\}$, because the magnitude of the field map in each voxel is usually less than $|\Delta f/2|$ [3]. Then, we use a regularized version of $\boldsymbol{\omega}^{(0)}$ as our initial field map for the iteration.

RESULTS: We applied the proposed method to a 1.5T knee data set with three echo times corresponding to relative fat-water phase shifts of $-\pi/6$, $\pi/2$, $7\pi/6$. Fig. 1 shows the initial field map, regularized estimated field map by the proposed method, and the resulting ML estimates of fat and water. Fig. 2 shows the field map estimated by the IDEAL method of [1] and the corresponding water and fat images. In this case the field inhomogeneity was fairly modest so the differences between the resulting fat and water images are subtle. (We will explore data sets with more severe field inhomogeneity.)

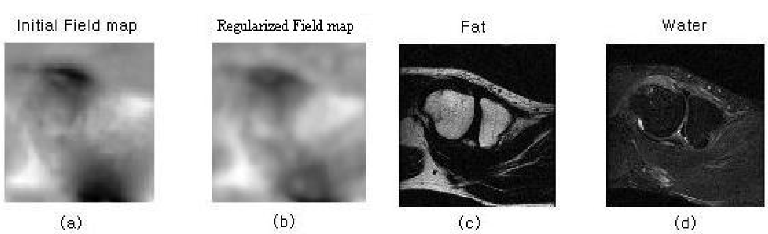

Fig.1. (a) Initial Field map, (b) Regularized estimated field map, (c) Fat, (d) Water.

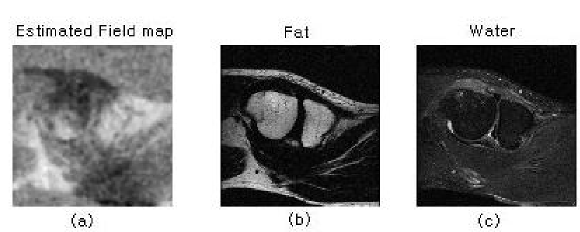

Fig.2. (a) Pixel independent method field map, (b) Fat, (c) Water.

CONCLUSION: We proposed a method for water-fat decomposition with regularized field map estimation. The experimental results show that regularization can lead to smoother field map estimates than the "pixel independent method" [1] as seen Fig. 2(a). Another potential advantage of the technique is avoiding a well known problem of fat-water swapping in pixels which contain just fat or water. To avoid such behavior, field map smoothness is used in heuristic region growing approach [3]. In the presented approach, incorporating roughness penalty into reconstruction may solve this problem automatically.

REFERENCES: [1] Reeder, *MRM*, 51(1): 35-45, 2004. [2] Reeder, *MRM*, 54(3): 636-44, 2005. [3] Yu, *MRM*, 54(4): 1032-9, 2005.