A Simplified Model for Stabilizing Alternating TR SSFP sequences

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Introduction Steady-state free precession (SSFP) signal amplitude oscillates when approaching steady state from thermal equilibrium, and causes artifacts in the images [1]. The long waiting period before the signal reaches steady state also compromises the effectiveness of important magnetization preparation schemes like fat saturation or inversion recovery, therefore an efficient initial preparation scheme is critical in non-continuous scans.

For conventional SSFP, there is a Fourier relation between RF flip angle increment and the resulted oscillatory residues. A Kaiser-Bessel windowed ramp catalyzation method was proposed to achieve a smooth and rapid SSFP signal transition [2]. Paired Kaiser-Bessel design has been shown to have a performance comparable to optimum SLR design when enough RF train length is allowed [3]. In this work we adapted the SU_2 formalism to build a simplified model for alternating TR SSFP (e.g., fat-suppressed ATR-SSFP[4], wideband SSFP[5]) sequences which justifies the use of Kaiser-Bessel windowed ramp, and developed a preparation scheme that can be applied to arbitrary TR_1/TR_2 and RF phase cycling combinations.

Theory Neglecting T_1 and T_2 relaxation [2], in an alternating TR sequence with TR_1/TR_2 and (0, π+φ) RF phase cycling, the magnetization transition matrix $\mathbf{R_0}$ for one full cycle (from TR_2 to TR_1, then back to TR_2) can be written as the product of a series of rotation matrices:

$$R_0 = [R_z(\theta_2)\, R_z(\varphi/2)\, R_x(-\alpha)\, R_z(-\varphi/2)\, R_z(\theta_1)]\,[R_z(\theta_1)\, R_z(-\varphi/2)\, R_x(\alpha)\, R_z(\varphi/2)\, R_z(\theta_2)] = R_2 R_1$$

where θ_1 (= $-\Delta f\times\pi\times TR_1$) and θ_2 (= $-\Delta f\times\pi\times TR_2$) are the amounts of phase offset during TRs, α is the flip angle. $\mathbf{R_1}$ can be mapped into its SU_2 form $\mathbf{Q_1}$:

$$\begin{aligned} Q_1 &= Q_z(\theta_1-\varphi/2)\, Q_x(\alpha)\, Q_z(\theta_2+\varphi/2) \\ &= \begin{bmatrix} e^{-i(\theta_1-\varphi/2)/2} & 0 \\ 0 & e^{i(\theta_1-\varphi/2)/2} \end{bmatrix} \cdot \begin{bmatrix} \cos(\alpha/2) & -i\sin(\alpha/2) \\ -i\sin(\alpha/2) & \cos(\alpha/2) \end{bmatrix} \cdot \begin{bmatrix} e^{-i(\theta_2-\varphi/2)/2} & 0 \\ 0 & e^{i(\theta_2-\varphi/2)/2} \end{bmatrix} \\ &= \cos(\alpha/2)\cos(\theta/2) - i \begin{bmatrix} \cos(\alpha/2)\sin(\theta/2) & \sin(\alpha/2)\, e^{-i(R\theta-\varphi)/2} \\ \sin(\alpha/2)\, e^{i(R\theta-\varphi)/2} & \cos(\alpha/2)\sin(\theta/2) \end{bmatrix} \end{aligned}$$

where $\theta = \theta_1 + \theta_2$, $R = (1-a)/(1+a)$, $a = TR_2/TR_1$. The angle (Ω) and axis ($\mathbf{n_1}$) of $\mathbf{Q_1}$ rotation can be directly extracted from the SU_2 matrix elements. Similar mapping can be done for $\mathbf{R_2}$. The value of Ω, the zenith angles of $\mathbf{n_1}$ and steady-state echoes are found to be proportional to flip angle α when α is small. Thus, after *p* excitations, the oscillatory residue ε_p perpendicular to the steady-state magnetization can be expressed as the Fourier transform of flip angle increment sequence $\{\Delta\alpha_k\ (= \alpha_k-\alpha_{k-1},\ \alpha_0 = 0,\ \alpha_p = \alpha)\}$:

$$\epsilon_p = \frac{1}{2}\left\{\frac{\sin[(R\theta-\varphi)/2]}{\sin(\theta/2)}\left[\sum_{k=1}^{p} e^{i\theta(k-1)}\,\Delta\alpha_k\right] + \frac{\cos[(R\theta-\varphi)/2]}{\cos(\theta/2)}\left[\sum_{k=1}^{p} e^{i(\theta-\pi)(k-1)}\,\Delta\alpha_k\right]\right\}$$

A Kaiser window sequence $\{w_k\}$ is then chosen as the flip angle increments for its good stopband attenuation. Next, we split $\{w_k\}$ into two series of its odd and even elements, multiply them with scalers b_1 and b_2 respectively ($b_1+b_2=1$ in order to meet the $\alpha_p=\alpha$ constrain), and re-write the oscillatory residue:

$$\epsilon_p = \frac{\sin[(R\theta-\varphi)/2]}{\sin(\theta/2)}\left[b_1\sum_{k=1}^{p/2} e^{i\theta(2k-2)} w_{2k-1} + b_2\sum_{k=1}^{p/2} e^{i\theta(2k-1)} w_{2k}\right] + \frac{\cos[(R\theta-\varphi)/2]}{\cos(\theta/2)}\left[b_1\sum_{k=1}^{p/2} e^{i\theta(2k-2)} w_{2k-1} - b_2\sum_{k=1}^{p/2} e^{i\theta(2k-1)} w_{2k}\right]$$

We can find optimized b_1 and b_2 for the center of passband θ_0 by making $\theta \sim \theta_0$. Minimizing $|\varepsilon_p|$ we have $b_1 = b_2 \times(1-R')/(1+R')$, $R' = \tan(R\theta_0-\varphi/2)/\tan(\theta_0/2)$. For example, considering wideband SSFP with $TR_2/TR_1 = a$, we found optimized b_1/b_2 for on-resonance spins is also *a* (Fig.1a). For fat-suppressed ATR SSFP with $TR_2/TR_1=1/3$, optimized $b_1=0$ and $b_2 = 1$ (Fig.1b).

Method Transient signal after linear ramp [6] and Kaiser ramp were measured in a uniform ball phantom. A linear shim was used to generate a frequency gradient. A pulse sequence was designed to optionally catalyze, then acquire the same phase encode for many imaging TRs. The process is repeated for all the phase encoding steps, so that a complete image can be reconstructed for each excitation during the approach to steady-state.

Results Figure 2 shows the measured transient signals from a uniform phantom, as functions of number of excitations and resonant frequency. Linear and Kaiser-Bessel ramps contained 6 cycles and were 43ms in length. Ripples were observed in the side-lobes when using linear ramp. Kaiser ramp created a better suppression of oscillatory residues and provides smooth signal transition.

Conclusion We introduced a simplified model for stabilizing alternating TR SSFP sequences. Transient signal fluctuation is reduced using proposed scaled Kaiser-Bessel ramp method. This design is easy to implement, and we expect it to be robust against B_1 variation.

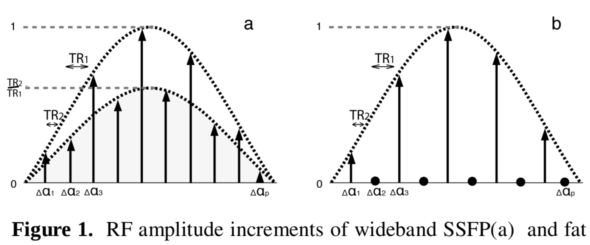

Figure 1. RF amplitude increments of wideband SSFP(a) and fat suppressed ATR-SSFP(b). Dotted profiles corresponds to a Kaiser-Bessel window with β= 3.

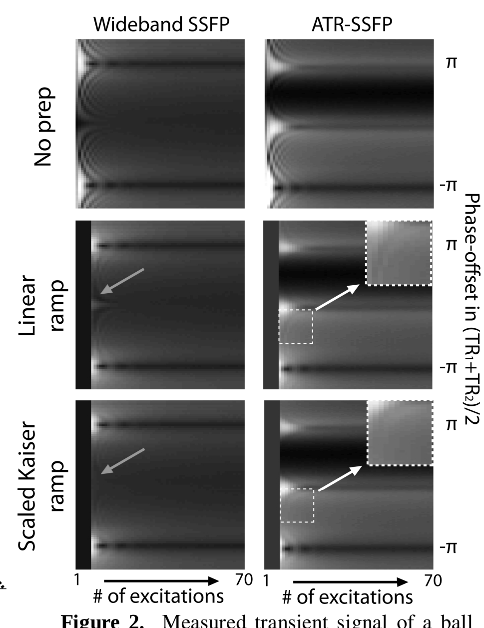

Figure 2. Measured transient signal of a ball phantom after different preparation methods. Left: wideband SSFP, gray arrows indicate the difference after linear ramp and scaled Kaiser ramp preparation. Right: ATR-SSFP.

References

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