An Improved Analytical Solution for Variable Density Spiral Design

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Introduction: Spiral MRI is a fast imaging technique that can be applied for a wide range of studies, such as cardiac imaging, functional brain imaging and flow measurement.(1, 2) To realize the real-time trajectory prescription on the scanner, analytic spiral *k*-space algorithms have been proposed.(1, 2) To fulfill the hardware constrains on the slew-rate and maximum gradient amplitude, the solutions normally are divided into two regions, that is, the slew rate limited region and the amplitude limited region. Further, either smooth transition function or exact solution is required to avoid the singularity at the *k*-space center. Recently, Kim and *et al.* (3) proposed an analytic solution for one class of variable density spiral design, which has recently been used in several studies, such as diffusion tensor imaging and spectroscopic imaging.(4) However, this trajectory design did not consider the singularity at *k*-space origin and thus could generate gradient trajectories with large gradient slew rate overflow. In this study, we demonstrated these potential pitfalls and proposed a simple analytical solution to this problem. The efficiency of our method will be illustrated using computer simulation and with phantom results.

Methods: The variable density spiral trajectory was expressed as,

$$k(\tau) = \lambda \tau^{\alpha} e^{j\omega\tau} \qquad [1]$$

Where τ is a function of time, $\omega = 2\pi n$ and n is the number of spiral turns. Given matrix size, N, field of view, FOV, $\lambda = N/(2 \times FOV)$. α determines the oversampling in the *k*-space. In the slew rate limited region, the solution is found by differentiating Eq. [1] twice with respect to time and neglecting the second term,

$$S(t) = \frac{\dot{\tau}^2}{\gamma}\frac{d^2k}{d\tau^2} + \frac{\ddot{\tau}}{\gamma}\frac{dk}{d\tau} \approx \frac{\dot{\tau}^2}{\gamma}\frac{d^2k}{d\tau^2} \qquad [2]$$

Where γ is gyromagnetic ratio. Setting $S(t)$ to a maximum available slew-rate value, S_m, and assuming $(\omega\tau/\alpha)^2 >> 1$, Kim and *et al.* got,

$$\tau(t) = \left(t/T_{es} \right)^{1/(\alpha/2+1)}, \quad T_{es} = \left(\left(\frac{\alpha}{2} + 1 \right) \sqrt{\frac{S_m \gamma}{\lambda \omega^2}} \right) \qquad [3]$$

For the new trajectory design, we obtained the solution by setting $S(t) = S_m\left(1 - e^{-t/L}\right)^2$,

$$\tau(t) = \left(\left(t + Le^{-t/L} - L\right) / T_{es} \right)^{1/(\alpha/2+1)} \qquad [4]$$

Where L is a parameter used to regularize the slew rate at original of the *k*-space. The new method is identical to Kim's solution if we set L equal to zero. Given the gradient raster time, Δt, L for a given trajectory was determined by taking the larger value of the solutions for following two cases. Case I, the slew rate for the first data point shall be less than S_m, we got $L_{case\ I} = \frac{\Delta t^2}{2T_{es}}\left(\frac{S_m \gamma \Delta t^2}{\lambda}\right)^{\frac{\alpha+2}{2\alpha}}$, which was derived using the approximation solution, $\tau(t) = \left(\frac{t^2}{2LT_{es}}\right)^{1/(\alpha/2+1)}$, with t → 0. Case II, let $S(t) = S_m/2$ for *P*-th data point, (*P* = 20 for this study), $L_{case\ II} = -\frac{P\Delta t}{\ln\left(1 - 1/\sqrt{2}\right)}$. We implemented both the new and the Kim's method in Matlab and also on the Siemens scanners. The equations for the gradient limited region were the same for both methods. The spiral trajectories were generated with the following parameters, Δt, 10 us, maximum slew rate, 150 mT/m/ms, maximum gradient amplitude, 24 mT/m, matrix size: 128×128, FOV: 256×256 mm^2. The *in vitro* data were acquired on a Siemens Tim 3.0T system with a 12-channel head coil for receiving and body coil for transmitting. The acquisition parameters were the same as above, except with slice thickness was 2.5 mm and TR/TE was 1000/4 ms.

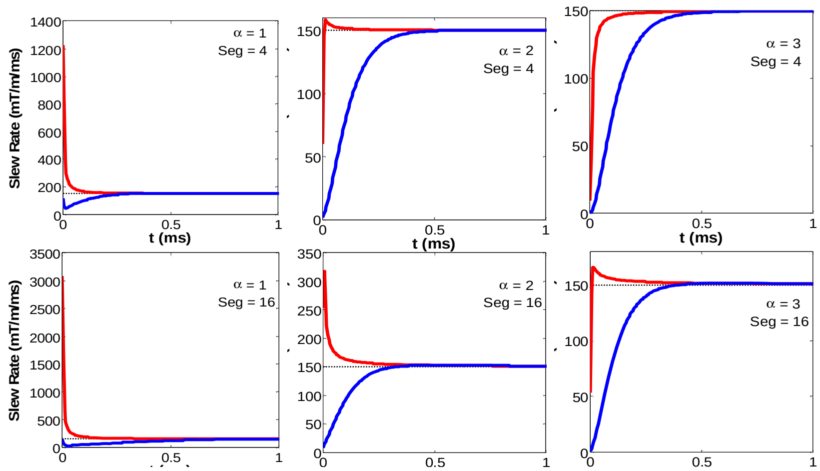

Fig. 1 The slew rates of the spiral trajectories. Red curve, Kim's spiral design. Blue curve, the new spiral trajectory design. Dotted line shows the targeted slew rate, 150 mT/m/ms.

Table 1 The maximum slew rates for the two trajectory designs

	The Maximal Slew Rates for the Trajectories (T/m/s)					
α/Seg	Kim's Spiral Design[3]			New Spiral Design		
	4	8	16	4	8	16
1.0	1225.8	1945.8	3088.8	151.7	151.5	150.6
1.5	292.6	406.4	751.5	150.7	151.7	153.9
2.0	158.8	216.4	317.4	150.2	151.0	152.6
2.5	150.0	156.3	206.4	150.0	150.6	151.9
3.0	149.9	150.5	165.5	149.9	150.3	151.5

Table 2 The total spiral duration for the two trajectory designs

	The Total Spiral Readout Durations (ms)					
α/Seg	Kim's Spiral Design[3]			New Spiral Design		
	4	8	16	4	8	16
1.0	14.05	7.03	3.51	14.13	7.15	3.76
1.5	17.68	8.74	4.27	17.76	8.83	4.35
2.0	20.40	10.03	4.84	20.48	10.11	4.92
2.5	22.52	11.03	5.28	22.60	11.12	5.36
3.0	24.21	11.83	5.63	24.29	11.92	5.72

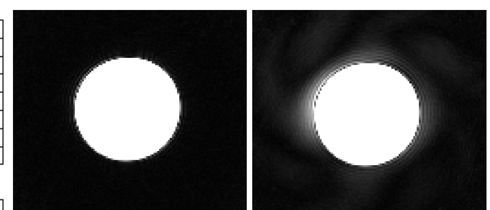

Fig. 2 The phantom images from the two spiral designs with α = 1.5 and 16 segments. Left, the new spiral designs, Right, the Kim's design.

Results and Discussion: Fig. 1 shows the slew rates within the first 1 ms for the two different spiral trajectory designs. The dotted lines showed the designed slew rate, 150 mT/m/ms. For the method described by Kim and *et al.* (red line in Fig. 1), the slew rates can be significantly higher than the targeted value, e.g., first figure in Fig. 1. Table 1 summarized more results of the maximum slew rates for the two methods with different α and spiral segments. Red fonts in Table 1 represented the trajectories with exceeded maximum slew rate (assuming a 5% tolerance). It indicates that the Kim's method tend to fail for spiral trajectory with small *k*-space oversampling factor or high spiral segments. On the other hand, the new method always gave solutions with the maximum slew rates only slightly differed from the targeted slew rate. Interestingly, Table 2 indicated that the total spiral readout duration of the new method only slightly increased (< 10%) for all cases. The main reason, as indicated in Fig.1, is that only the initial small portion of spiral trajectory needs to be modified so that the maximum slew rate does not exceed the target value. Fig. 2 shows the phantom results on the Siemens scanner with a hardware gradient slew rate hardware limit of 400 mT/m/ms. For α = 1.5 and 16 segments, the Kim's method will lead to an inaccurate trajectory at the *k*-space center since its maximum slew rate is 752 mT/m/ms (see Table 1). Because of this, the phantom results (right, Fig. 2) showed a "spiral arms" artifact outside the phantom and blurring at the phantom edges. On the contrary, the phantom image from the new trajectory design does not show significant artifacts.

References: [1] Glover, G. H., *et al.*, Magn Reson Med 1999; **42**:412-415. [2] Duyn JH., *et al.*, J Magn Reson 1997; **128**:130-134. [3] Kim D-H., Magn Reson Med 2003; **50**:214-219. [4] Liu C., *et al.*, Magn Reson Med 2004; **52**:1388-1396.