Design Metrics for Data Undersampling and Weighting Strategies

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Introduction: Undersampling in Non-Cartesian MRI can decrease scan time but produces incoherent aliasing. Data weighting also affects both aliasing and resolution[1]. Comparing trajectory and weighting schemes requires objective metrics; a framework for such metrics is the goal of this work. The proposed method is illustrated using variable density spiral trajectories[2] with four undersampling schemes (Fig. 1), and two weighting schemes[1] to either **(I)** minimize aliasing error (reducing resolution), or **(II)** maintain full resolution.
Method: Gradient specs, # interleafs, FOV, and ADC time should be equal for all trajectories (here gradients = 40/150, 15 interleafs, FOV = 24cm, such that the ADC duration for full sampling at 1 mm resolution is 10 ms). Eight trajectory/weighting schemes were designed to achieve ADC durations of 5, 6, and 7 msec (some methods could not reasonably achieve the shorter ADC durations).

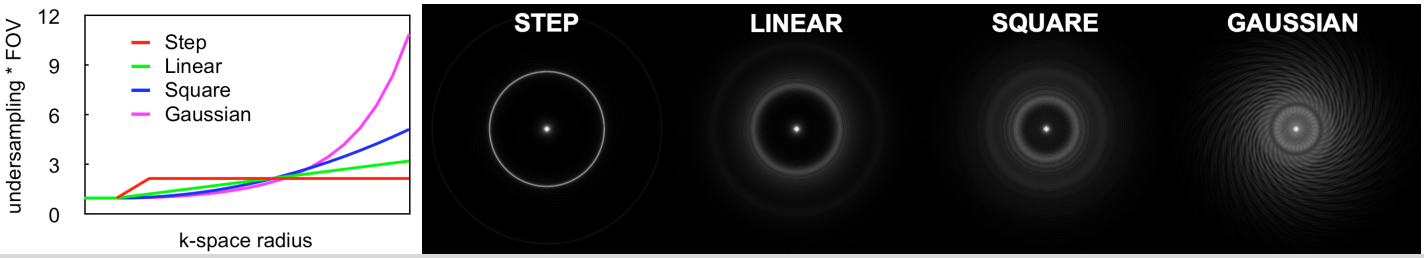

Fig. 1. Illustration of VD-spiral undersampling schemes (graph) and corresponding PSF's windowed (identically) to show aliasing. These PSF's are transformed from the MTF's weighted using scheme II (see Intro) and undersampled to achieve ADC = 5 msec.

1. Resolution Normalization: Resolution is affected by the width of k-space sampled (MTF width) and the applied data weights (MTF shape). For min-aliasing weights **(I)**, the prescribed resolution was < 1mm so that the true resolution after weighting was 1 mm. True resolution was fixed by keeping constant both the height (k=0) and area of the *smoothed* MTF, which by definition holds constant the area and height of the PSF mainlobe, respectively (Fig. 2). **All trajectories given here resulted in a true resolution of 1mm.**

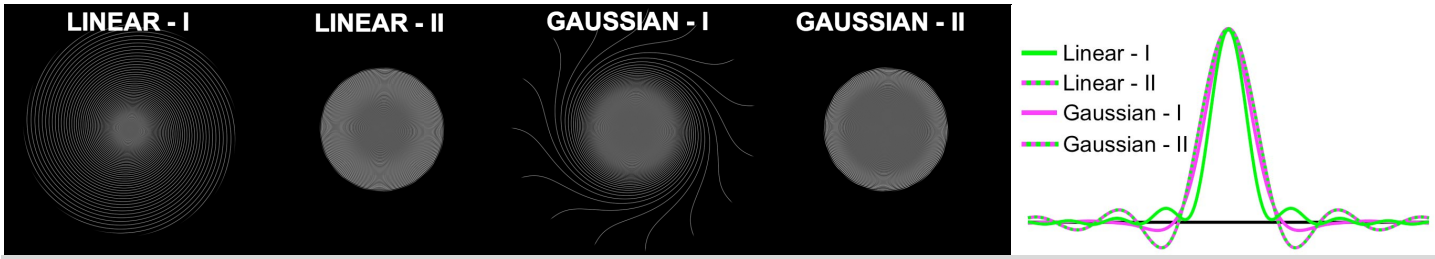

Fig. 2. Linear and Gaussian undersampling MTF's with weighting schemes (I) and (II) for ADC = 6 ms (linear) and 7 (Gaussian) ms, and resulting PSF mainlobes. Areas of mainlobes are equal; this is a good metric for resolution when comparing different shaped PSF's.

2. Relative SNR Metric: Sampling density correction decreases SNR by nonuniformly weighting data. Neglecting T2* decay, N data points weighted by $\{w_n\}$ will have a relative SNR due to weighting given by[3] $SNR_{relative} = \Sigma(w_n) / [N \Sigma(w_n^2)]^{-1/2}$.

3. Proposed Aliasing Metric: The PSF is calculated by gridding only $\{w_n\}$ (i.e. data = "1") onto a Cartesian k-space grid using the prescribed sampling pattern and taking the FFT. If the k-space Cartesian grid spacing is less than 0.5/FOV, the transform gives the full PSF (2 FOV wide); examples are shown in Fig. 1. The PSF outside of the central (fully supported) region reflects the aliasing lobes; these are tapered to reflect the average contribution to the FOV, and aliasing metric (Fig. 3) is equal to its energy.

Maximum Undersampling / Prescribed Resolution

Method	ADC = 5 ms	ADC = 6 ms	ADC = 7 ms
Step I		2.9 / .75 mm	1.5 / 1.0 mm
Step II	2.2 / 1.0 mm	1.8 / 1.0 mm	1.5 / 1.0 mm
Linear I		8.2 / 0.6 mm	2.1 / 0.9 mm
Linear II	3.2 / 1.0 mm	2.4 / 1.0 mm	1.9 / 1.0 mm
Square I			4.8 / 0.8 mm
Square II	5.2 / 1.0 mm	3.4 / 1.0 mm	2.4 / 1.0 mm
Gaussian I			1000 / 0.5 mm
Gaussian II	10.9 / 1.0 mm	4.9 / 1.0 mm	2.8 / 1.0 mm

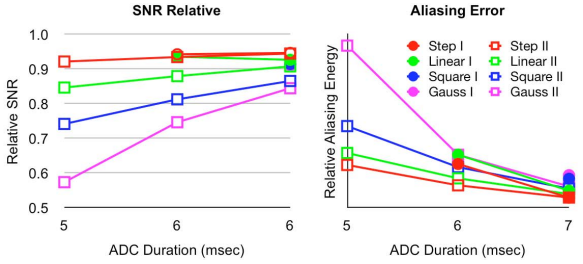

Fig. 3. Parameters used to obtain desired ADC duration and final resolution of 1 mm (table), and graphs of resulting SNR and aliasing metrics. Gray boxes in table indicate a prescribed resolution of < 0.5 mm was required for a final resolution of 1 mm, and the solution was not calculated.

Conclusions: This work presents a general method for comparing undersampled trajectories. Other aliasing metrics which reflect aliasing structure may be more desirable than that proposed here for some applications, but fit into the proposed framework for normalization. In its present form, this work considers only single coil reconstruction; extension to multiple coil reconstruction, and "parallel imaging" reconstruction, will be pursued. Specific to VD-spiral, this work also suggests that data should be weighted to minimize error over the FOV that can be supported ("weighting scheme II"), and that trajectories which undersample more consistently over k-space produce the best SNR and have the least aliasing energy (i.e. step best, then linear, square, and Gaussian worst).
References: 1. Mag Res Med 43(6): 867-75. 2. Mag Res Med 43(3): 452-8. 3. Mag Res Med 34(2): 170-8.