Quantifying the Signal to Noise Ratio Benefit of Apodization by Sampling Density Design: A Demonstration with Sodium MRI of the Human Brain

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Introduction: Especially for low signal applications where large voxels must be incorporated to generate acceptable image SNR, Gibbs' ringing can have a significant detrimental impact on image quality. For sodium imaging of the human brain, the banding artefact may cover much of the brain (**Figure 1**). This artefact can be minimized with post-acquisition apodization, but a better approach is to generate an inherent apodization filter with k-space sampling density design (**1**). The purpose of this abstract is to quantify the SNR advantage of apodization by sampling density design and demonstrate it with sodium imaging of the human brain.

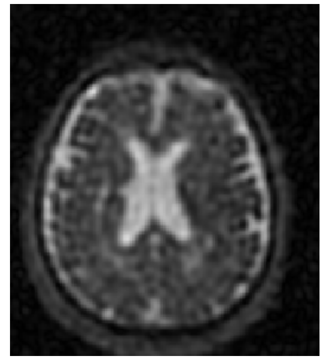

Figure 1: A sodium image generated with uniform k-space acquisition highlighting the effect of Gibbs' ringing on image quality.

Theory and Methods: Consider the case where M samples have been acquired for every k-space location. This 'over-sampling' can be redistributed so that the k-space sampling density is a scaled version of the desired apodization filter. Upon normalization, the modulation transfer function (MTF), and by corollary the image signal and resolution, will be equivalent for the case of uniform k-space acquisition with post acquisition filtering (UPF), and the case of sampling density filtering (SDF). However, it can be shown that the noise variance in the image is reduced in the SDF approach (**Equations 1-5**). This SNR advantage was demonstrated with 3D twisted projection acquisition (**2**), developed to produce a 'generalized-Hamming' sampling density filter (**Equation 6**, with n = 1, and m = 0.16). 3700 projections, fully sampling k-space with 832500 sampling points, were implemented for both an SDF case and a uniform sampling density case (multiplied by the same 'generalized-Hamming' filter post-acquisition). For both cases, the readout duration (17.9 ms) and k-space extent sampled (139 1/m) were the same along with the number of averages acquired (N = 2) and the scan duration (just over 3 minutes). All images were acquired on a Varian Inova 4.7T scanner with a TR of 25 ms, an RF pulse length of 900 μs, and a flip angle of 55°.

$$\text{(1)}\quad P_{UPF}(k) = \frac{F(k)^2}{M}\sigma_S^2$$

$$\text{(2)}\quad P_{SDF}(k) = \left(\frac{\sum_K F(k)}{MK}\right) F(k)\sigma_S^2$$

$$\text{(3)}\quad \sigma_{I(UPF)}^2 = \sigma_S^2 \frac{1}{MK}\sum_K F(k)^2$$

$$\text{(4)}\quad \sigma_{I(SDF)}^2 = \sigma_S^2 \frac{\left(\sum_K F(k)\right)^2}{MK^2}$$

$$\text{(5)}\quad \frac{\sigma_{I(SDF)}^2}{\sigma_{I(UPF)}^2} = \frac{\left(\sum_K F(k)\right)^2}{K\sum_K F(k)^2}$$

Equations (1,2): Noise power spectral density for both the UPF and SDF cases when the MTF has been normalized so its value at the center of k-space is one.

Equations (3,4): Noise variance in each image, from the noise power spectral density

Equations (5): Relative image noise variance.

F(k) is the apodization filter; K is the total number of locations in k-space; and σ_s^2 is the noise variance associated with each data point

$$\text{(6)}\quad F(r) = s_1 - s_2\cos\left(\pi(1+r)\right)$$

$$s_2 = \frac{[1/m - n]}{\left[1-\cos\left[\pi(1+m)\right]\right]}$$

$$s_1 = s_2 + n$$

Equation (6): 'Generalized Hamming' filter used in this study

Results and Discussion: From **Equation 5** it can be predicted that a 17% SNR advantage is associated with the use of SDF over UPF for the apodization filter implemented. This advantage was demonstrated in three volunteers (**Figure 2**). The SDF approach also has a noise colouring advantage, as the noise power reduction occurs at lower spatial frequencies. **Equation 5** is very similar to that derived in reference (**3**) for the relative noise variance associated with sampling density compensation. These equations highlight that it is more SNR efficient to generate a desired MTF (in the case of sodium MRI an apodizing shape) with sampling density rather than with post-acquisition processing.

References: **(1)** Greiser, A., von Kienlin, M., MRM 50, 1266 (2003) **(2)** Boada, F.E, et. al., MRM 38, 1022 (1997) **(3)** Pipe, J.G., Duerk, J.L., MRM 34, 170 (1995)

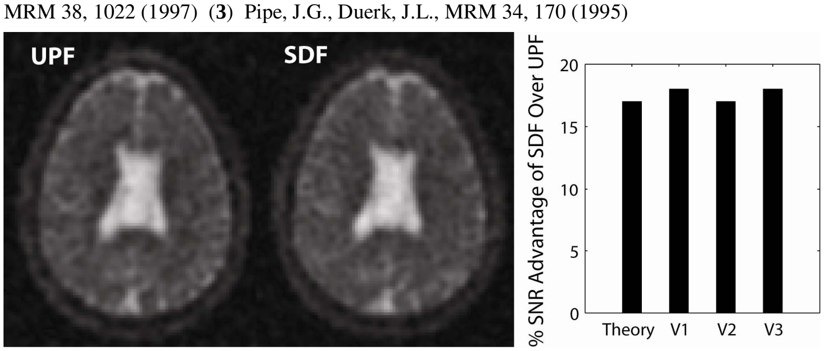

Figure 2: The relative SNR advantage of apodization by sampling density filtering (SDF) over uniform k-space acquisition with post-acquisition filtering (UPF). This advantage is demonstrated on images a healthy volunteer, and quantified for three volunteers (V1, V2, V3).