The Optimal Acquisition Strategy for Exponential Decay Constants Estimation.

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INTRODUCTION. Estimating the relaxation constant of an exponentially decaying signal from experimental MR data is fundamental in diffusion tensor imaging, fractional anisotropy mapping, measurements of transverse relaxation rates and contrast agent uptake. An exponentially decaying signal (Eq. [1]) can be described by two parameters: its amplitude ρ and decay rate λ, where t is the user-controlled encoding parameter (diffusion weighting or echo time, *etc.*). Therefore, at least two measurements with different encodings t_1 and $t_2 > t_1$ are needed to estimate λ. It has been shown that within all such "two-point" schemes, the imaging time is used most efficiently when $(t_2 - t_1)$ is chosen to be about 1.29/λ (1). The method also requires that the number of averages of the acquisition with encoding parameter t_2 to be about 18/5 the number of averages with t_1.

$$S(t) = \rho e^{-\lambda t} \qquad [1]$$

$$A_{i\bullet} = \frac{1}{\sigma_0}\left(e^{-\lambda t_i} \qquad -t_i \rho e^{-\lambda t_i}\right) \qquad [2]$$

In this report we demonstrate numerically that the most efficient multi-point schemes (with several, more than two encodings) are not more efficient than the two-point one with the same total measurement duration.

THEORY. In a multi-point acquisition N data points, S_i, are collected, each with its own encoding parameter t_i. (We assume $t_1 \le t_2 \le t_3 \le \ldots \le t_N$ with the equal sign allowing repetition of acquisitions with the same encoding.) We also assume that all N acquisitions are subject to Gaussian noise with standard deviation σ_0 determined by the receiver hardware, spatial resolution and sample properties. Then, the optimal acquisition strategy is specified by a set of t_i's yielding the highest precision in the decay rate, *i.e.* the smallest variance σ_λ^2 given by the second diagonal element of the 2×2 covariance matrix $(A^+A)^{-1}$ of the χ^2 fitting procedure. The rows of the N×2 matrix A are given by Eq. [2]. We perform this optimization in the vicinity of the expected decay value $\lambda = \lambda_{tune}$ and then evaluate σ_λ at arbitrary λ's.

METHODS. It is clear that optimal value of t_1 is zero otherwise the amplitude ρ can be redefined as $\rho\, e^{-\lambda t1}$ and all t_i's as $(t_i - t_1)$. After this redefinition, ρ is the signal with minimal exponential weighting (*i.e.* signal acquired with minimal echo time or diffusion weighting allowed by the sequence/hardware) and t_1=0. The remaining (N-1) values of t_i were optimized numerically using a brute-force method in (N-1) dimensions on a uniform grid of $\lambda_{tune}\Delta t$ = 0.05 between 0 and 4. The size of the search grid is dictated by the flattening of the signal dependence at large λt, Eq. [1], while its granularity — by the numerical complexity. The optimal values of t_i for N between 2 and 10 are given in table 1. Computation of the last line took 8 hours on a Pentium IV, 3.4 GHz PC. The dependence of σ_λ on λ using several optimal N-point acquisition strategies is shown in Fig. 1.

N	$\lambda_{tune}t_i$									
2	0	1.10								
3	0	1.20	1.20							
4	0	1.25	1.25	1.25						
5	0	1.30	1.30	1.30	1.30					
6	0	1.35	1.35	1.35	1.35	1.35				
7	0	0	1.20	1.20	1.20	1.20	1.20			
8	0	0	1.25	1.25	1.25	1.25	1.25	1.25		
9	0	0	1.25	1.25	1.25	1.25	1.30	1.30	1.30	
10	0	0	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.30

Table 1. *Optimal encoding parameters t_i for estimating decay constant λ in the vicinity of λ_{tune} for several N-point methods. Note that all protocols (except N=9) are found to be "two-point" ones. The N=9 case appears different due to the sampling grid granularity: two points out of the three lie on the neighboring grid marks.*

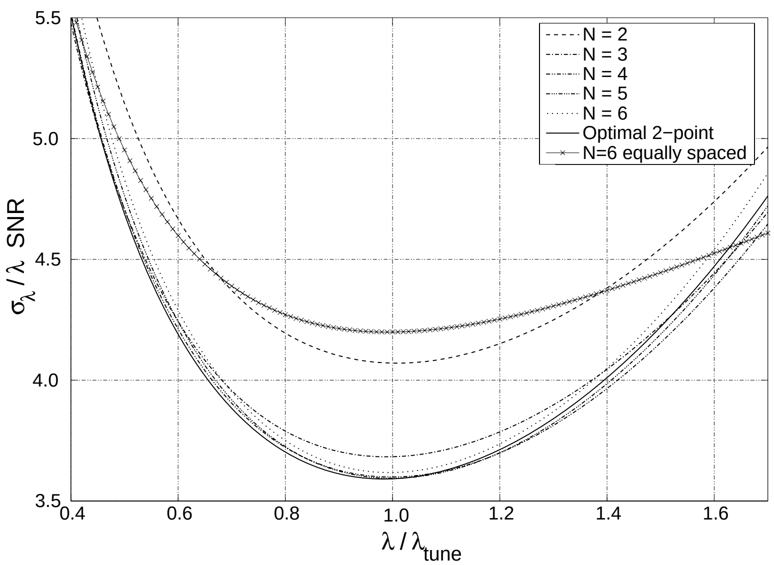

Fig. 1. *Precision as a function of λ for several N-point protocols tuned for λ_{tune}. Here, the SNR is the signal-to-noise ratio of the average of N images with minimal exponential weighting and in general is a function of λ if the minimal weighting is not zero. Note that the precision of protocols with N > 3 is already very close to optimal. Since the convexity of the curves is very small, the precision stays within 15% of the best over a broad interval 0.6 < λ/λ_{tune} < 1.5 . As an example of a commonly used scheme, the precision for optimal 6-equally-spaced-point protocol is superposed.*

CONCLUSION. The optimal N-point protocols presented in table 1 reveal that within the search grid all schemes (except N=9) are, in fact, two-point methods. The N=9 case appears different due to the sampling grid granularity: two points 1.25 and 1.30 out of the three lie on the neighboring grid marks. Performing the search on a finer grid to validate this is computationally intractable. Therefore, we conjecture that the highest precision of exponential relaxation estimation is achieved using the optimal two-point method (1). The hardware, sample and duration independent measure of this precision can be characterized by the dimensionless normalized coefficient of variation $\varepsilon_\lambda = (\sigma_\lambda/\lambda)\times\text{SNR}$= 3.6 (1) where SNR is the signal to noise ratio obtained by averaging N acquisitions with minimal exponential weighting, $S(t_1)$. For example, if averaging N such images delivers SNR of 100, then the relative error in the decay constant estimation can not be smaller than σ_λ/λ=3.6/100=3.6%.

References. (1) Fleysher L. et al., Magn Reson Med 2007;57:380.