SLR RF Pulse Design for Arbitrarily-Shaped Excitation Profiles

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Introduction: The Shinnar-Le Roux (SLR) algorithm has been proven tremendously useful to design large flip-angle pulses, but its application has been limited to the excitation of rectangular profiles [1]. In such cases, the pulse design problem reduces to a well-known digital filter problem. However, there is no intrinsic limitation to rectangular profiles. In this work, we introduce two modifications to the SLR algorithm to achieve accurate excitation profiles in both magnitude and phase. We demonstrate the accuracy of the algorithm by designing large flip-angle RF pulses producing wavelet-shaped excitation profiles. Indeed, this method might be very useful to increase SNR in wavelet encoding, which has been so far restricted to small flip-angle excitation [2] except in specific situations [3, 4].

Methods: The SLR algorithm reduces the RF pulse design problem to the design of two polynomials $A_n(z)$ and $B_n(z)$. The excitation slice profile is $M_{xy}^{+} = 2\alpha^{*}\beta$, where α and β are the Cayley-Klein parameters. For a rotation by an angle φ around the y axis, $\alpha = \cos(\varphi/2)$ and $\beta = \sin(\varphi/2)$, so $M_{xy}^{+} = \sin(\varphi)$. Therefore, we need $\beta = \sin\{\arcsin(M_{xy}^{+})/2\}$. That is the first modification we propose. We then branch back to the classical SLR algorithm. We approximate β by a polynomial $B_n(z)$ using an inverse Discrete Fourier Transform (DFT). $A_n(z)$ is the corresponding minimum-phase polynomial. It is first obtained as described in [1]. An additional step is then needed to eliminate any phase distortion in the resulting profile. We precompensate the phase that α will have, correcting β after zero-padding [5]: $\hat{\beta}_{precompensated} = \hat{\beta}e^{i\angle\hat{\alpha}}$, where ^ denotes the DFT and $\angle\hat{\alpha}$ takes the phase of $\hat{\alpha}$. That is the second modification. The inverse DFT of $\hat{\beta}_{precompensated}$ gives the new β. The corresponding minimum-phase α is then generated.

Results: Figure 1 compares the desired and excited magnetization profiles M_{xy}, when the desired excitation profile is a degree 3 Battle-LeMarie wavelet of width 2^{-3} [6]. The excited profile is almost perfect. Notice that to be visible, the real part of the magnetization has been multiplied by a factor 100. Figure 2 shows the corresponding RF pulse. As we have mentioned, the perfect refocusing requires zero-padding, which lengthens the RF pulse on both sides. For specific applications, this step might not be required, if only the magnetization magnitude is of interest. However, it is crucial in wavelet encoding implementation.

Conclusion: We have derived a new RF pulse design method for accurately exciting arbitrarily-shaped profiles. The method builds on the SLR algorithm and inherits its simplicity and strength.

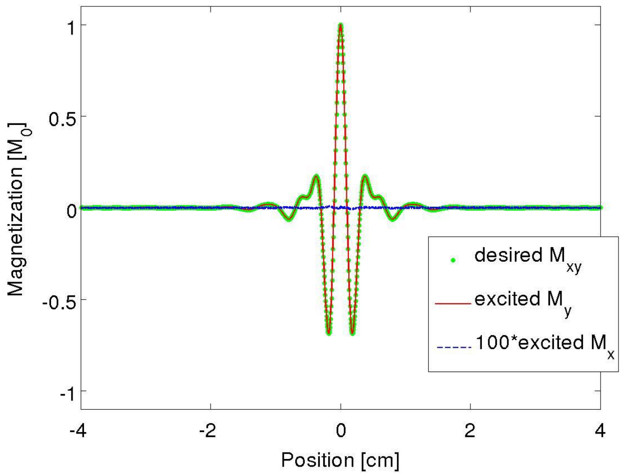

*Fig. 1: Desired (dotted green) and excited (red) profiles. There is almost no remaining phase (blue: 100*real part).*

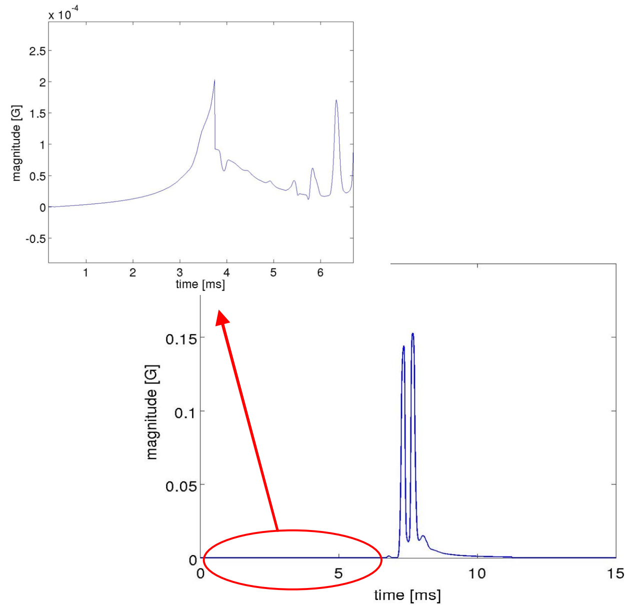

Fig. 2: Corresponding RF pulse.
The zoomed-in region shows the lengthening of the pulse due to the zero-padding of β. It is necessary to control the phase and get perfect refocusing.

References:
1. Pauly, J.M., et al., IEEE Trans. Med. Imaging, 1991. **10**(1): p. 53-65.
2. Panych, L.P., P.D. Jakab, and F.A. Jolesz, J Magn Reson Imaging, 1993. **3**(4): p. 649-55.
3. Serrai, H. and L. Senhadji, J Magn Reson, 2005. **177**(1): p. 22-30.
4. Xu, Z., Magn. Reson. Imaging. , 2004. **22**(1): p. 47-54.
5. Zhu, L. and J.M. Pauly, in *ISMRM, Fourteenth Scientific Meeting and Exhibition*. 2006. Seattle, WA. p.2995
6. Mallat, S., *A wavelet tour of signal processing* 1999: Academic Press.