Equiripple Design of Multidimensional RF Pulses via Convex Optimization

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INTRODUCTION: We propose a new approach to designing RF pulses which achieves equiripple excitation profile. It formulates the RF design problem as a convex optimization problem. Thus, other physical constraints (e.g. peak RF) and regularization terms (e.g. RF power) can be easily embedded into the problem if they are represented in the form of convex functions. This approach can be extended to parallel RF transmission.

THEORY: The convex optimization technique was introduced to find the optimal coefficients in a filter design problem [1]. Lu et al. showed that the SDP-based method, a subclass of convex optimization, can be useful in designing optimal 2-D filters [2]. If the system is linear, which is true in filter design and within the small-tip-angle (STA) regime [3], the problem can be approximated by defining finely enough discrete input (**x**) and desired output (**d**), whose relationship is represented by a system matrix (**A**). We adapted this approach for RF pulse design and it can be categorized into two methods based on the way **A** is defined.

$$\mathbf{y} = \mathbf{A}\mathbf{x},\ \mathbf{e} = \mathbf{d} - \mathbf{y} \text{ where } \mathbf{x} \in R^{N\times 1},\ \mathbf{y} \in R^{M\times 1}$$

- **Method 1**: DFT (SLR transformation required [4]) ⇨ $x_n = f(t_n)$, $y_m = F(\omega_m)$ [2]
- **Method 2**: STA approximation ⇨ $x_n = B_1(t_n)$, $y_m = M(r_m)$ [5]

The optimization problem can be stated as finding the optimal input ($\mathbf{x}^*$) whose output ($\mathbf{y}^*$) is the closest to the desired profile (**d**) in the sense that it minimizes the Chebyshev norm of the error vector (**e**) and we solve this via convex optimization with the aid of solvers like SDPT3 [7]. **Method 1** is an indirect mapping between the RF pulse ($B_1(t)$) and the magnetization profile (M(r)) but it can handle the non-linearity of the Bloch equation through the SLR transformation [4]. In contrast, **Method 2** directly maps the RF pulse to the magnetization profile but works only for a STA excitation.

$$\begin{aligned} &\text{minimize} \quad \delta \\ &\text{subject to} \quad w_m^2 |e_m|^2 \le \delta \quad for\, m = 1,2,\ldots,M \end{aligned}$$

$$w_m^2 |e_m|^2 \le \delta \iff \delta - w_m^2\left(e_{R,m}^2 + e_{I,m}^2\right) \ge 0 \iff \begin{bmatrix} 1 & 0 & w_m e_{R,m} \\ 0 & 1 & w_m e_{I,m} \\ w_m e_{R,m} & w_m e_{I,m} & \delta \end{bmatrix} \ge 0$$

Note that the optimization achieves equiripple error in passband and stopband by equalizing the complex amplitude of the weighted error vector. Also, we can control the relative ripple size by adjusting the weighting vector (**w**). For example, we can set **w** to a constant to have equiripple size everywhere. Even though the objective function represents a simple ripple size, we can easily combine any other convex function such as l_2-norm to include regularization since convexity is preserved. Also, we can easily specify more constraints to accommodate additional physical constraints such as peak RF and RF power.

METHODS: As a feasibility test, we designed and simulated 3 representative design examples. Linear-phase pulses with different transition widths are compared and different ripple sizes were obtained as in Fig. 1. A quadratic-phase pulse example [8] is given to show that an arbitrary phase profile can be used as a target profile (Fig. 2). 2-D spatial excitation pulse with variable-density spiral excitation *k*-space is designed in the same way, but by solving a larger problem than 1-D case (Fig. 3). When designing RF pulses, we used CVX [9], a MATLAB®-based modeling system, to formulate the optimization problem, and the results were fed into the simulator (T_1, T_2 ignored) [10] to verify the excitation profiles.

RESULTS AND DISCUSSION: We demonstrated the possibility of utilizing pre-developed convex optimization solvers in designing RF pulses, specifically for an equiripple excitation profile. This is advantageous since it finds an optimal solution and is ready for including additional object functions and constraints in convex form, which simplifies the RF pulse design problem.

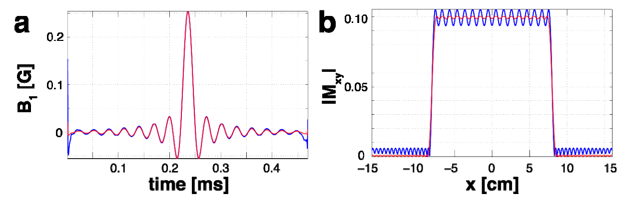

Figure 1. Linear-phase pulses via **Method 2** for different transition widths (blue: 5mm, red: 10mm) (a) RF pulses (θ = 0.1 rad) (b) simulated excitation profiles

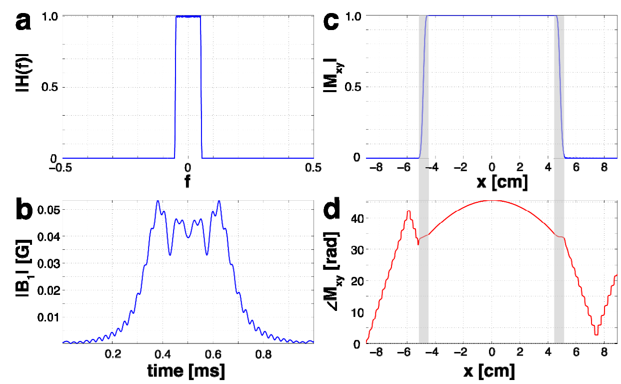

Figure 2. Quadratic-phase pulse via **Method 1** (n = 400, f_p = 0.0475, f_s = 0.0550, k = 120, grey bands indicate "don't care" region) (a) designed filter (b) RF pulse (θ = π/2 rad) (c,d) simulated excitation profile

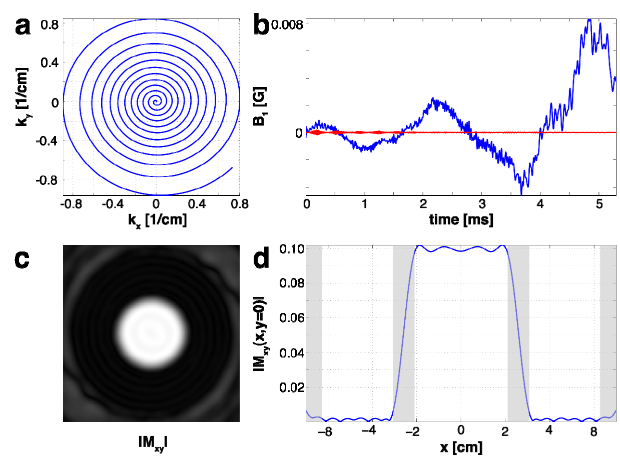

Figure 3. 2-D spatial excitation pulse via **Method 2** (a) excitation *k*-space (b) waveforms (θ = 0.1 rad - blue: $B_{1,x}$, red: $B_{1,y}$) (c,d) simulated excitation profile

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