Toeplitz-based acceleration of RF pulse design for parallel excitation

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Introduction

Recently, an iterative algorithm for designing the RF pulses in the spatial domain for parallel excitation [1] was proposed, which showed that the multi-dimensional parallel excitation problem in a small-tip angle regime could be solved efficiently using a Conjugate Gradient(CG) algorithm together with Non-Uniform FFT(NUFFT) operations. This abstract proposes a method to further accelerate the computation time of the aforementioned method using Toeplitz matrices and a CG algorithm. This method eliminates the need for interpolation and in certain circumstances, can reduce the dimensionality of the minimization problem.

Theory

The small-tip RF design method of [1] minimizes the following cost function iteratively :

$$\arg\min_{\mathbf{b}} \quad \|\mathbf{d}-\mathbf{A}\mathbf{b}\|^2 + \beta\|\mathbf{b}\|^2, \qquad \mathbf{A} = [\mathbf{S}_1\mathbf{E} \;\; \mathbf{S}_2\mathbf{E} \;\; \ldots \;\; \mathbf{S}_L\mathbf{E}]$$

where d : a desired pattern, E : a small-tip angle Fourier encoding matrix, S_i : a diagonal matrix representing the coil sensitivity for the ith coil, b : a stack of the RF pulses for each coil, β : a regularization parameter that controls the RF pulse power.

The solution of this minimization problem is $\mathbf{b} = (\mathbf{A}'\mathbf{A} + \beta\mathbf{I})^{-1}\mathbf{A}'\mathbf{d}$.(' denotes Hermitian transpose.) Applying push-through identity for matrices to this expression yields the alternative expression, $\mathbf{b} = \mathbf{A}'(\mathbf{A}\mathbf{A}' + \beta\mathbf{I})^{-1}\mathbf{d}$ where $\mathbf{A}\mathbf{A}' = \sum_{i=1}^{L}\mathbf{S}_i\mathbf{E}\mathbf{E}'\mathbf{S}_i'$ and $\mathbf{E}\mathbf{E}'$ is a Toeplitz matrix if we assume that RF pulses are short enough to disregard the magnetic field inhomogeneity. In this reformulated problem, we first solve for $\mathbf{m} = (\mathbf{A}\mathbf{A}' + \beta\mathbf{I})^{-1}\mathbf{d}$ by the CG algorithm and then compute the RF pulses from $\mathbf{b} = \mathbf{A}'\mathbf{m}$. The benefit of this approach is that we need to perform only FFT operations in order to evaluate the product between a Toeplitz matrix(EE') and a column vector [2] in the CG algorithm because this product can be rewritten as a convolution. On the other hand, in the CG algorithm of the conventional method [1], NUFFT operations [3] are performed and it involves both FFT and interpolation. As pointed out in [2], each iteration in the CG algorithm of this approach is expected to be twice as fast as the conventional method. Furthermore, in certain circumstances, the size of $\mathbf{A}\mathbf{A}'$ smaller than that of $\mathbf{A}'\mathbf{A}$, reducing the dimensionality of the minimization problem.

Results and Discussion

To compare the performance between the conventional NUFFT method and Toeplitz method, we computed 2D RF pulses for a smoothed rectangular excitation pattern with both methods and performed a Bloch equation simulation. The field of view was 24cm by 24cm where 64 by 64 grid was applied. We used a spiral k-space trajectory of length 2.9ms with 8 transmission coils and a parallel imaging speedup factor of 2.4. On a computer with Intel Pentium 4, 3.2 GHz CPU, 2GB RAM and Matlab 7.2, it took the conventional NUFFT method 12.9 sec to compute the RF pulses while it took the Toeplitz method 6.9 sec to compute the RF pulses for 100 iterations in the CG algorithm. We see from simulated excitation patterns in the Fig.1, Fig. 2 and Fig. 3 that the performance of the Toeplitz method is equivalent to that of the NUFFT method. Though the purpose of β is to control the RF pulse power, it affected the convergence rate of this algorithm. The proper choice of β to balance the excitation error, RF power deposition and the convergence rate is a topic for further investigation.

Conclusion

In this abstract, we presented a method based on Toeplitz matrices to further accelerate the iterative RF pulse design method for parallel excitation.

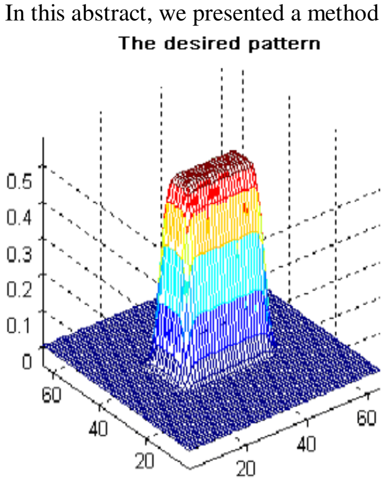

Fig.1. The desired pattern

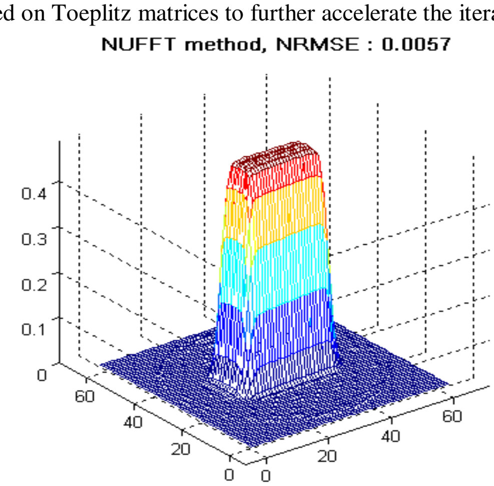

Fig.2. The excitation pattern by the method [1]

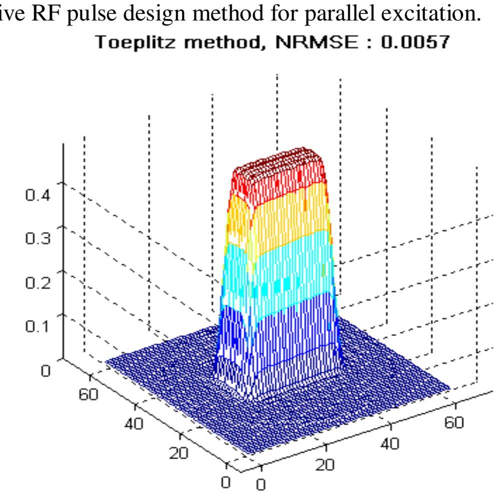

Fig. 3. The excitation pattern by the Toeplitz method

References

[1] Grissom, *Mag. Res. Med.*, 56(3):620-9, Sep. 2006. [2] Fessler, *IEEE Trans. Sig. Proc.*, 53(9):3393-402, Sep. 2005. [3] Fessler, *IEEE Trans. Sig. Proc.*, 51(2):560-74, Feb. 2003