Sparsity-Enforced Coil Array Mode Compression for Parallel Transmission

A. C. ZELINSKI[1], V. ALAGAPPAN[2], V. K. GOYAL[1], E. ADALSTEINSSON[1,3], AND L. L. WALD[2,3]

[1]RESEARCH LABORATORY OF ELECTRONICS, MIT, CAMBRIDGE, MA, UNITED STATES, [2]MARTINOS CENTER FOR BIOMEDICAL IMAGING, HARVARD MEDICAL SCHOOL, MGH, LONGWOOD, MA, UNITED STATES, [3]HARVARD-MIT DIVISION OF HEALTH SCIENCES & TECHNOLOGY, CAMBRIDGE, MA, UNITED STATES

INTRODUCTION. Parallel transmit (pTX) systems reduce the duration of spatially-tailored excitation pulses by utilizing the array's spatial transmit patterns to allow under-sampling of excitation *k*-space [1]. Hardware costs and complexity, however, limit the number of channels. Forming linear combinations of array elements can transform the spatial modes of the array into a different basis set, potentially capturing a majority of the TX efficiency and acceleration capabilities in a subset of the modes. The available TX channels are then applied only to a subset of array modes, which are chosen based on their contribution to TX efficiency and encoding. When designing & choosing *reception* modes, the object being imaged is unknown and the choice of array modes must be based on general sensitivity and encoding considerations. In the TX case, however, one knows and in fact chooses the desired excitation pattern, so explicit incorporation of this knowledge into the selection of the mode subset may be useful. Here we propose a fast *target-dependent* sparsity-enforced subset selection (SESS) algorithm that explicitly accounts for the desired excitation pattern when choosing the mode subset, in contrast with principal component or covariance analysis methods that use only the spatial profiles and determine only a *single* subset for all excitations [2]. Here, we use SESS to determine 8-mode subsets of a 16 mode array for use on an 8-ch pTX system at 7T when forming slice-selective spatially-tailored excitations with uniform and ring-shaped target patterns in a water phantom [3]. Excitations using the SESS subsets are compared to those chosen from covariance analysis of the mode spatial maps. Brute-force search finds that for this coil, SESS actually finds the best of all 12,871 possible subsets for the uniform excitation, and a near-optimal subset for the ring-shaped one.

Fig. 1: Channel Modes, Water Phantom at 7 Tesla

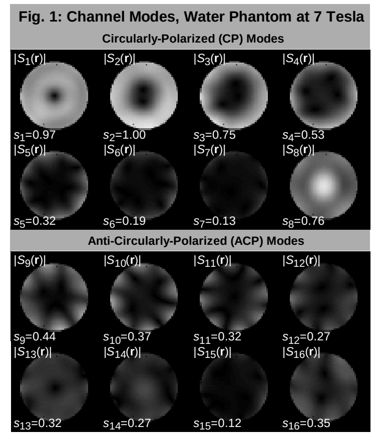
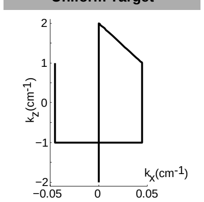


Fig. 2: k-Space Trajectory, Uniform Target



METHODS. Fig. 1 shows the 16 TX modes (S_n) of a stripline array in an orthogonal birdcage (BC) basis configuration, as obtained via a Butler matrix [4]. Modes include those with correct polarization for excitation (CP modes) and those with the opposite polarization (ACP modes).

Spoke-based pulses. Spoke pulses are comprised of weighted sinc-like segments in k_z placed at different locations in (k_x, k_y) using an echo-volumnar trajectory [3]. The sincs excite a slice in *z*, while (k_x, k_y) weights tailor the in-plane excitation into a chosen pattern. Fixing the placement of *T* spokes, along with spoke shape and gradients ($\mathbf{G}(t)$), ends up fixing the slice-selective properties of the pulse. Users are then free to shape the in-plane pattern by choosing the weights that each of the *P* modes deposits at each of the *T* locations.

Sparsity-Enforced Subset Selection (SESS). SESS is an optimization that designs a set of waveforms to drive the *N* modes to form the target pattern while imposing a strong penalty whenever a mode waveform becomes nonzero. This penalty enforces sparsity on the number of modes and reveals a small subset of good modes & corresponding waveforms. Here we derive SESS for *T*-spoke trajectories of duration *L*, where spoke locations are fixed at $\mathbf{k}_1, \ldots, \mathbf{k}_T$, and $\mathbf{k}(t)$ is known $\forall$ time *t*. To begin, assume that all *N* modes may be driven. The equations that relate the pulses used to drive each mode to the resulting magnetization $m(\mathbf{r})$ are linearized [5] to yield $\mathbf{m}=\mathbf{S}_1\mathbf{F}\mathbf{g}_1 + \cdots + \mathbf{S}_N\mathbf{F}\mathbf{g}_N = \mathbf{A}_{\text{tot}}\mathbf{g}_{\text{tot}}$ (Eq. 1), where $\mathbf{m} \in \mathcal{C}^{Ms}$ contains samples of $m(\mathbf{r})$ at $\mathbf{r}_1, \ldots, \mathbf{r}_{Ms}$, $\mathbf{S}_n \in \mathcal{C}^{Ms\times Ms}$ is a matrix of $S_n(\mathbf{r})$ samples, $\mathbf{g}_n \in \mathcal{C}^T$ contains weights mode *n* places at *T* spoke locations, $\mathbf{F}(u,v)= j\gamma\Delta_t\cdot M_0\cdot\exp(j\mathbf{r}_u\cdot\mathbf{k}_v)\cdot\exp(j\Delta B_0(\mathbf{r}_u)(t_v\text{-}L))$, $\mathbf{F} \in \mathcal{C}^{Ms\times T}$, $\mathbf{A}_{\text{tot}} = [\mathbf{S}_1\mathbf{F} \cdots \mathbf{S}_N\mathbf{F}]$, & $\mathbf{g}_{\text{tot}} = [\mathbf{g}_1^T \cdots \mathbf{g}_N^T]^T$. Eq. 1 thus describes the in-plane excitation arising when the modes deposit energy at spoke locations. The desired excitation, $d(\mathbf{r})$, is vectorized to $\mathbf{d} \in \mathcal{C}^{Ms}$. Solving $\mathbf{d}=\mathbf{A}_{\text{tot}}\mathbf{g}_{\text{tot}}$ (Eq. 2) for $\mathbf{g}_{\text{tot}}$ via the pseudoinversion of $\mathbf{A}_{\text{tot}}$ will indeed reveal good weightings, but results in a solution where *all* modes deposit energy, i.e., all $\|\mathbf{g}_n\|_2$s are nonzero, failing to reveal a useful small mode subset. Consider however solving Eq. 2 while penalizing each nonzero mode energy (each $\|\mathbf{g}_n\|_2$), explicitly prohibiting the use of many modes, while encouraging those that do remain in use to have $\mathbf{g}_n$s that still approximately solve Eq. 2. This is accomplished by solving $\min_{\mathbf{g}\text{tot}} \{\|\mathbf{d} - \mathbf{A}_{\text{tot}}\mathbf{g}_{\text{tot}}\|_2^2 + \lambda\cdot\sum_n(\|\mathbf{g}_n\|_2)\}$ (Eq. 3) for fixed λ, where $\sum_n(\|\mathbf{g}_n\|_2)$ is the L_1-norm of the $\|\mathbf{g}_n\|_2$ mode energies; such a norm encourages sparsity [6]. As λ:0→1, increasing numbers of modes have their energies driven to zero, residual error increases, and smaller subsets of modes & weightings are revealed. SESS differs from [7] because here we enforce sparsity on modes rather than on spoke locations. For reasonable *T*, finding a SESS subset takes only several seconds. After solving Eq. 3 using a large λ, those *P* modes with the largest $\|\mathbf{g}_n\|_2$s are chosen as the optimized *P*-mode subset.

Pulse design. For a fixed *P*-mode subset, weights are tuned by truncating the *N* - *P* non-chosen terms from $\mathbf{A}_{\text{tot}}$ & $\mathbf{g}_{\text{tot}}$ & least-squares fitting to Eq. 2.

Fig. 3: Mode Subsets & Resulting Excitations, Uniform Target

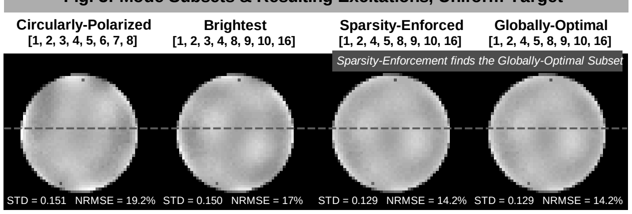
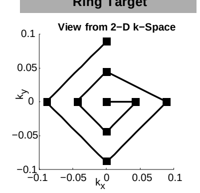
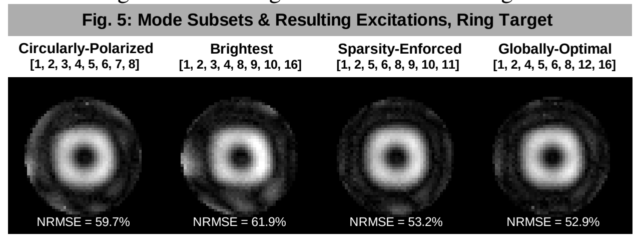


Fig. 4: k-Space Trajectory, Ring Target



RESULTS. **Uniform excitation**. The target is a uniform excitation and a 3-spoke trajectory is used (Fig. 2). Excitation quality is evaluated across four subsets: 8 CP modes, 8 brightest modes, 8 SESS-optimized modes (λ = 0.3), and 8 globally-optimal (GO) modes. The latter is found via brute-force search over all (16 choose 8) subsets. For each subset, weightings and 3-spoke pulses are designed and undergo Bloch simulation. Figure 3 shows that the reasonable choice of CP modes produces the worst excitation. Choosing the 8 bright modes improves NRMSE by 1.13x, but fails to reduce σ. The SESS modes ($Sp(\mathbf{r})$, $p \in\{1,2,4,5,8,9,10,16\}$) produce a noticeably more uniform excitation; σ & NRMSE improve by 1.16x and 1.20x relative to the bright mode result. The bright subset makes use of $S_3(\mathbf{r})$ (with L_2-energy 0.75), whereas SESS opts to use $S_5(\mathbf{r})$ (with L_2-energy 0.32): using a "dark mode" in lieu of a bright one results in a better excitation. Brute-force search over all mode subsets shows that SESS determines the best subset among all (16 choose 8) = 12,871 possible.

Fig. 5: Mode Subsets & Resulting Excitations, Ring Target



Ring-shaped excitation. For the ring-shaped target pattern and 9-spoke trajectory (Fig. 4), choosing the bright modes produces the worst excitation. SESS modes ($Sp(\mathbf{r})$, $p \in\{1,2,5,6,8,9,10,11\}$) outperform both CP and bright modes, producing a 1.12x lower-error excitation relative to CP. SESS's subset here differs from the GO one, but leads to essentially the same result: the GO excitation has only 1.006x lower NRMSE. The GO and SESS subsets do *not* equal those of the uniform case: different targets call for the use of different modes. Further, dark modes are present in both the SESS and GO solutions (e.g., each chooses $S_6(\mathbf{r})$, the 2nd darkest of all 16 modes).

CONCLUSION. SESS finds non-obvious mixtures of light and dark modes, providing increases in excitation quality relative to CP and bright modes. Furthermore, SESS exploits knowledge of the desired target excitation pattern and produces target-specific mode subsets.

REFERENCES. [1] Katscher et al. MRM03;49:144-150. [2] Buehrer. ISMRM05, p. 2668. [3] Saekho et al. MRM06;55(4):719-724. [4] Alagappan et al. MRM06;57(6):1148-1158. [5] Grissom et al. MRM06;56(3):620-629. [6] Chen et al. SIAM Rev01;43(1):129–159. [7] Zelinski et al. ISMRM07, p 1691.