Fast Optimal Control Method for Large-tip-angle RF Pulse Design in Parallel Excitation

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Introduction The optimal control method [1,2] is among existing techniques for large-tip-angle RF pulse design in parallel excitation [1,3-5]. Optimal control has some advantages over other methods; it is robust, it applies to general excitation trajectories, and it provides control over the excited phase pattern. However, it requires long computation times, rendering it unsuitable for online pulse design. We introduce a *fast* optimal control method that achieves the same accuracy as optimal control in a shorter time, and with reduced memory requirements.

Theory The proposed fast optimal control method is based on a linearization of the spin-domain Bloch equation about an initial set of large-tip-angle pulses $\{b_1,\ldots,b_R\}$, where R is the number of coils. It is derived by subtracting the Bloch equation for $\{b_1,\ldots,b_R\}$ from that of a *perturbed* pulse set $\{b_1+\tilde{b}_1,\ldots,b_R+\tilde{b}_R\}$, where $\{\tilde{b}_1,\ldots,\tilde{b}_R\}$ are perturbation pulses we will design. This yields differential equations for the perturbations $(\tilde{\alpha}^*,\tilde{\beta})$ to the initial pulses' spinor parameters (α_0^*,β_0) caused by the perturbation pulses. Approximating cross terms between $(\tilde{\alpha}^*,\tilde{\beta})$ as zero decouples their solutions, and focusing on $\tilde{\beta}$ (a symmetric analysis applies to $\tilde{\alpha}$), we have:

$$\tilde{\beta}(\boldsymbol{x},T) \approx \frac{i\gamma}{2}\sum_{r=1}^{R} s_r(\boldsymbol{x}) \int_0^T \tilde{b}_r^*(t)\,\alpha_0^*(\boldsymbol{x},t)\,e^{\frac{i\gamma}{2}\boldsymbol{x}\cdot\int_t^T \boldsymbol{G}(s)ds}dt, \qquad (1)$$

where s_r is coil r's transmit sensitivity and T is the pulse length. For pulse design, we would like to evaluate (1) rapidly using non-uniform fast Fourier transforms (NUFFT's) [6]. Though it contains a Fourier kernel, (1) is non-Fourier due to $\alpha_0^*(\mathbf{x},t)$, which, when discretized, becomes a high-rank matrix $\boldsymbol{\alpha}_0$. To enable fast computation, we transform $\alpha_0^*(\mathbf{x},t)$ to a frame rotating at the frequency induced by the gradients, and absorb the transforming complex exponential into the Fourier kernel. In the gradient frame $\boldsymbol{\alpha}_0$ has low rank, and can be accurately approximated as:

$$\boldsymbol{\alpha}_0^* \approx \sum_{l=1}^{L} \boldsymbol{c}_l \boldsymbol{d}_l' = \boldsymbol{C}\boldsymbol{D}', \qquad (2)$$

using small L. To obtain $\boldsymbol{D'}$, we compute the SVD of $\boldsymbol{\alpha}_0^*$ for a small subset of spatial locations. We then compute the least-squares optimal coefficients $\boldsymbol{C}$ for all spatial locations using a running sum during Bloch simulation, obviating the need to store the full $\boldsymbol{\alpha}_0^*$ matrix. In matrix/vector form:

$$\tilde{\boldsymbol{\beta}} = \sum_{r=1}^{R} diag\{s_r(\boldsymbol{x}_i)\} \sum_{l=1}^{L} diag\{c_{il}\}\,\boldsymbol{G}diag\{d_{lj}\}\,\tilde{\boldsymbol{b}}_r, \qquad (3)$$

where $\boldsymbol{G}$ is an NUFFT operator. We use $\tilde{\beta}$ and $\tilde{\alpha}$ to calculate the perturbed magnetization. To design $\{\tilde{b}_1,\ldots,\tilde{b}_R\}$ we form a least-squares cost function in terms of the magnetization and minimize it using a linear Conjugate Gradient (CG) algorithm.

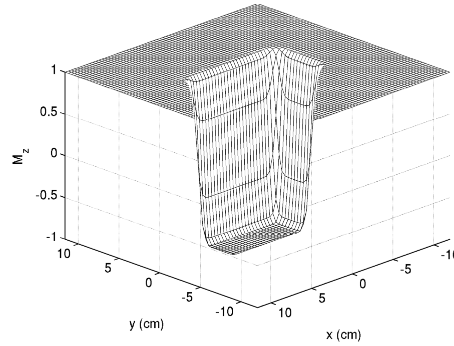

Figure 1. *Desired inversion pattern.*

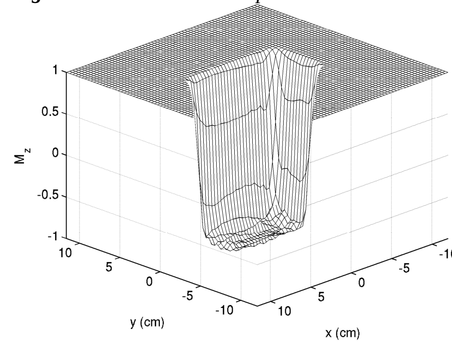

Figure 2. *Inversion pattern excited by the fast optimal control-designed pulses, NRMSE = 0.0918.*

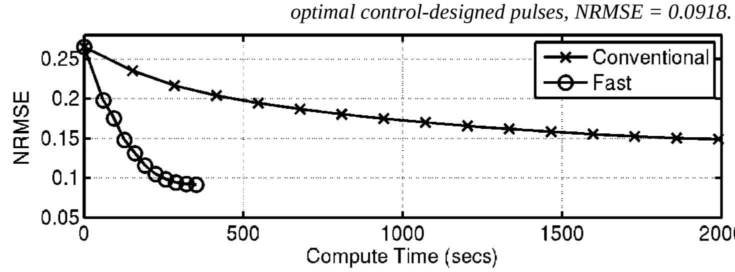

Figure 3. *Measured design times for conventional and fast optimal control methods. The fast method achieves a 37-fold reduction in compute time.*

We designed echo-planar (EP) inversion pulses using both methods for an 8-channel head array [7] in MATLAB 7.2 (Mathworks, Natick, MA, USA) on a 3.4GHz PIV PC with 2Gb RAM. The desired inversion pattern (Fig. 1) was a smoothed 10 x 5 cm rectangle, with FOV 24cm and resolution 0.375 cm. The EP trajectory had resolution 0.625 cm and XFOV = 8 cm, (speedup factor = 3, T = 4.6 ms). The fast method alternated between 25 CG iterations and a Bloch simulation to update the (α_0^*,β_0) approximations, which used L=4. Conventional optimal control used gradient descent, with a step size chosen to maximize the decrease in error per iteration while avoiding divergence. Normalized RMS error (NRMSE) vs. compute time was recorded. Both methods were initialized with small-tip-designed pulses [8].

Results Figure 2 shows the longitudinal magnetization excited by the fast optimal control-designed pulses. NRMSE for these pulses is 0.0918. Figure 3 shows that to reach this error, conventional optimal control required much more time than the fast method; 238 minutes compared to 6.5 minutes, or 37 times longer. Furthermore, efficient implementation of conventional optimal control requires storage of 6 N_s x N_t matrices, where N_s is the number of spatial points, and N_t is the number of RF pulse samples per coil. Storage of these matrices can be prohibitive for longer pulse lengths and for 3D pulse design. In contrast, the fast method only requires storage of NUFFT interpolators, 2 N_s x L matrices, and 2 N_t x L matrices.

Conclusion We have introduced a fast optimal control method for parallel excitation, and demonstrated its ability to produce pulses of the same quality as conventional optimal control in a much shorter time, and with smaller memory requirements. The fast method uses the efficient linear CG algorithm to design pulses. Though pulse design with the fast method still required several minutes, the method presents several opportunities to accelerate computation, e.g. by computing NUFFT's in parallel, and by parallelizing Bloch simulations temporally or spatially.

References [1] D Xu et al. 16th ISMRM, p1696, 2007. [2] S Conolly et al. IEEE TMI, 5:106-115, 1986. [3] J Ulloa et al. 15th ISMRM, p3016, 2006. [4] K Setsompop et al. 16th ISMRM, p677, 2007. [5] W Grissom et al. 16th ISMRM, p1689 2007. [6] J Fessler et al. IEEE Trans Sig Proc, 51:560-574, 2003. [7] S Wright. 10th ISMRM, p854, 2002. [8] W Grissom et al. *MRM*, 56(3):620–9, 2006. This work supported by NIH Grant DA015410.