

# Regularized Non-Cartesian SENSE Using a Multiscale Wavelet Model

B. Liu<sup>1</sup>, E. Abdelsalam<sup>2</sup>, and L. Ying<sup>1</sup>

<sup>1</sup>University of Wisconsin - Milwaukee, Milwaukee, WI, United States, <sup>2</sup>GE Healthcare, waukesha, WI, United States

## INTRODUCTION:

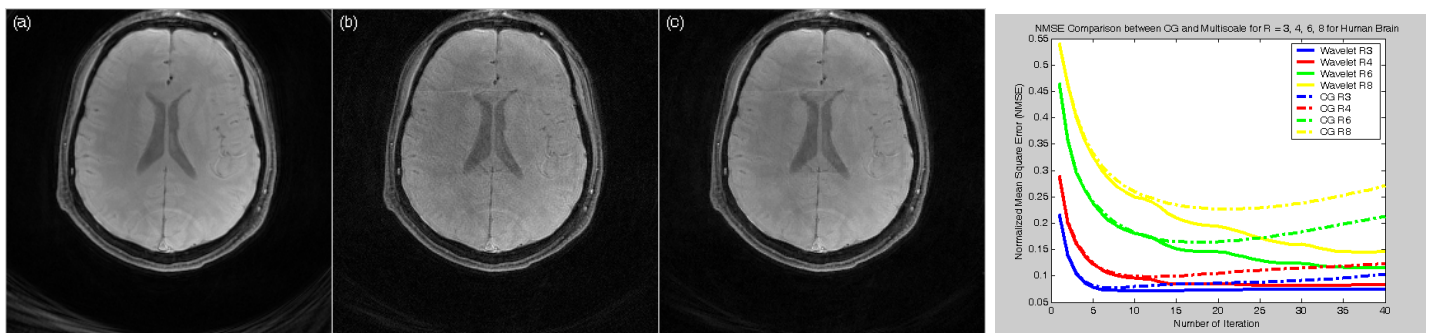
Many advances have been made for SENSE regularization with Cartesian trajectories (1). In contrast, very few have been made for non-Cartesian ones. The iterative conjugate gradient method (CG-SENSE) (2) has been widely used for non-Cartesian SENSE. However, when the reduction factor is large, the ill-conditioning problem prevents the CG-SENSE from converging to the optimal reconstruction (3). To address this problem, we propose a regularization technique where the regularization term is a multiscale wavelet transform. The method has the advantage that the wavelet coefficients at different wavelet scales can be weighted by different regularization parameters such that both the smooth regions and sharp edges of images can be represented accurately (4). The experimental results demonstrate that the proposed method improves the convergence behavior and the reconstruction quality.

## THEORY:

In CG-SENSE, the least squares solution to the imaging equation  $m = \mathbf{E}v$  [1] (with  $v$  the desired image,  $m$  the downsampled  $k$ -space data,  $\mathbf{E}$  the sensitivity encoding matrix as in (1)) is obtained by solving  $v = \arg \min_v (\|m - \mathbf{E}v\|_2)$  [2] using iterative CG method. Here, we model the image as a generalized Gaussian random field, which has been shown to accurately represent the smooth and textured regions as well as preserving edges in image processing applications (4). Under this model, the reconstructed image is given by  $v_{\text{reg}} = \arg \min_v (\|m - \mathbf{E}v\|_2 + \|\Lambda \Psi v\|_p^p)$  [3] where  $\Psi$  denotes the wavelet transform matrix at all desired scales,  $\Lambda$  is a diagonal matrix whose elements are regularization parameters at different scales, and  $\|\cdot\|_p$  denoted the  $L_p$  norm ( $0 < p \leq 2$ ). The proposed regularization method takes advantage of the fact that the wavelet transform coefficients of medical images are sparse, and can be intuitively interpreted as minimizing the sparsity of the image in wavelet domain under the data consistent constraint. In choosing the regularization parameter in  $\Lambda$ , we let the weight for the scaling coefficient be  $\lambda_1$  and those for the wavelets coefficients be  $\lambda_2 2^{-\alpha j}$  at scale  $j$  (4), where  $\lambda_1$ ,  $\lambda_2$ , and  $\alpha$  are constants determined heuristically. Due to the nonlinearity of the above convex optimization, fixed point algorithm (5) was used to solve Eq. [3].

## METHOD AND RESULTS:

In vivo human data have been acquired on a GE 3T scanner (GRE sequence, coil Num = 8, TE = 3.5ms, TR = 2s, flip angle = 90, FOV = 24cm, slice Thickness = 4.0mm, spacing = 1.0mm, matrix size = 256\*256, interleave num = 24, points/interleaf = 2332, repetition = 10, spiral out). To simulate the reduced acquisition in parallel imaging, we manually selected 3, 4, 6, 8 out of 24 interleaves, corresponding to reduction factors R= 8, 6, 3, 4. The parameters were set as  $p=1$ ,  $\alpha=1.2$ ,  $\lambda_1=0.1$  and  $\lambda_2=0.2$  in our implementation. We show the reconstruction at R = 4 after 40 iterations in Fig. 1 (c). For comparison, the images from SoS (sum of square) and CG-SENSE at R = 4 after 20 iterations (corresponding to the best result as shown in Fig. 1(d)) are shown in Fig. 1(a) and (b), respectively. We compared the convergence behaviors of the proposed method with CG-SENSE in Fig. 1(d) where the normalized mean squared errors (NMSE) were plotted as a function of the number of iterations at different reduction factors. It is seen that the proposed method effectively addresses the converge-and-then-diverge issue of CG-SENSE, and achieves a much lower level of error, especially at high reduction factors (R=6 and 8). In terms of computational complexity, the proposed method is about the same as CG-SENSE. Each iteration for both methods takes 4.2 seconds on a 2.8GHz CPU/512MB RAM PC.



**Fig. 1:** Reconstruction of in vivo human brain acquired with 8 coils, spiral trajectory, and  $R = 4$ . (a) SoS reconstructed from fully sampled data, (b) CG-SENSE without regularization, (c) Proposed regularized reconstruction (d) Convergence curve at different reduction factors.

## DISCUSSION AND CONCLUSION:

The proposed method is able to address the semi-convergence problem of CG-SENSE and the final reconstruction is improved. The reconstruction is shown to significantly reduce the artifacts in CG-SENSE reconstruction, especially at a large reduction factor.

## REFERENCES:

[1] Lin F-H, et al. MRM 51: 559-567(2004) [2] Pruessmann KP, et al. MRM 46: 638-651 (2001). [3] Qu P, et al. MRM 54: 1040-1045 (2005). [4] Belge M, et al. IEEE Tran. Image Processing, 9: 597-608 (2000) [5] Vogel CR, SIAM 2002.