

# Quantification of SNR and $g$ -Factor for Parallel MRI: Universal Application to Image-Based and $k$ -Space-Based Image Reconstructions

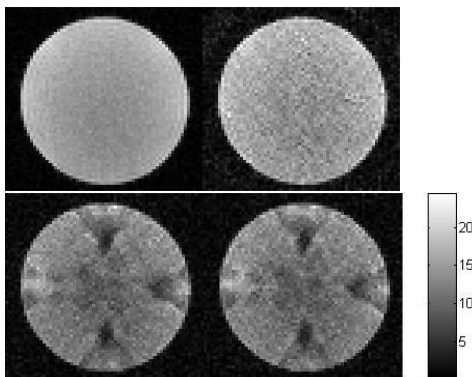
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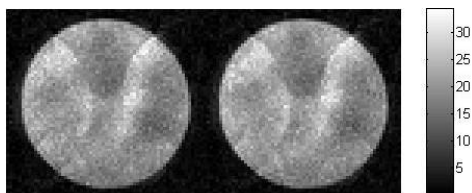
**Introduction:** In this work we develop a general and robust method for calculating image noise, SNR, and  $g$ -factor that will allow quantification of  $g$ -factor for parallel imaging where existing methods have failed. Analytical approaches (1) calculate directly the image noise and the  $g$ -factor, and also produce images in units of SNR (2). However, these methods are limited by two factors: i) the image noise matrix formalism (1) may only be used for linear reconstruction algorithms for which a reconstruction matrix may be written; ii) calculation of image noise directly from the reconstruction matrix becomes computationally unfeasible as the reconstruction matrix increases in size (3). Generalized SENSE reconstruction techniques conform to the linear reconstruction. GRAPPA-like techniques conventionally include a non-linear root-sum-of-squares step. However the image reconstruction may be made linear by the “complex-GRAPPA” approach (4) which uses the (Nyquist-sampled) Auto-Calibration Signal lines as a coil sensitivity reference for complex image combination. The reconstruction matrix is typically too large to permit direct noise calculation in the following cases: SENSE with Cartesian variable-density 2-D undersampling; SENSE with arbitrary  $k$ -space trajectories; GRAPPA with 2-D acceleration or with 2-D or 3-D reconstruction kernels. The pseudo multiple replica SNR measurement is a simple Monte Carlo method (5) which emulates the acquisition of multiple images. This method requires a linear reconstruction algorithm in order to operate on image and noise signals independently. In this work, we combine the pseudo multiple replica method with linear reconstruction algorithms for both generalized SENSE and GRAPPA reconstruction techniques for cases in which direct image noise analysis is unfeasible.

**Methods:** The pseudo multiple-replica SNR method emulates acquisition of image replicas by adding correctly scaled and correlated signal noise to the acquired  $k$ -space data before image reconstruction. This is repeated multiple times using the same “image”  $k$ -space and newly synthesized noise (5). From the pseudo images noise may be calculated (as the standard deviation through the stack of the real or imaginary part of the complex pixel values) on a pixel-by-pixel basis and hence allows the mapping of spatially-variant SNR and  $g$ -factor for parallel imaging reconstructions. Synthetic noise must have the same level and correlation of signal noise received by the elements of the array-coil. This is measured in a separate “noise pre-scan” (2) when the receiver is opened with no normal NMR signal present and with the coils loaded by the object as for imaging. Image noise maps are calculated from the stack of pseudo image replicas. SNR maps are formed from dividing the original image by the noise map.  $g$ -Factor maps are formed from the noise map calculated for the accelerated data and an equivalent image noise map for an unaccelerated acquisition. Linearity of the image reconstruction allows reconstruction of purely synthetic noise of a fully-sized dataset (i.e. without any undersampling or acquired ‘image’ signal). To define  $g$ -factor correctly care must be taken to find the optimal image noise for the unaccelerated data by including knowledge of the level and correlation of noise in the receiver-elements in the image reconstruction. Linear image reconstruction algorithms were used for generalized SENSE (1):  $\rho = (\mathbf{E}^T \Psi^{-1} \mathbf{E})^{-1} \mathbf{E}^T \Psi^{-1} \mathbf{s}$  where  $\rho$  is the vector of image pixels,  $\mathbf{E}$  is the encoding matrix,  $\Psi$  is the sample noise matrix, and  $\mathbf{s}$  is the vector of  $k$ -space data; and for GRAPPA (4):  $\rho = \mathbf{C} \mathbf{H} \mathbf{T} \mathbf{s}$  where  $\mathbf{C}$  is a coil combining matrix containing SNR-optimal complex coil sensitivity data,  $\mathbf{H}$  is a Fourier transform matrix, and  $\mathbf{T}$  is the transfer matrix containing weights used to linearly combine neighboring  $k$ -space data to reconstruct missing data. Image reconstruction was implemented both directly as matrix operations for SENSE and GRAPPA (with the matrix inverse for SENSE) or iteratively for generalized SENSE or by many convolution-like operations (as is conventional) for GRAPPA.

**Results:** The pseudo multiple replica SNR measurement was rigorously validated in phantom images by comparison to SNR measurements from multiple acquired image replicas and analytical image noise calculations (1, 2). Figure 1 shows an image reconstructed with a generalized SENSE method from data undersampled in two dimensions by a factor of 2 in each with a densely sampled center of 8 lines giving a total acceleration factor  $R = 3.1$ , together with the corresponding SNR maps calculated using actual image replicas and the pseudo multiple replica method. Figure 2 shows SNR maps for the image reconstructed from the same undersampled data using a GRAPPA technique. Figure 3 shows an image and corresponding  $g$ -factor map *in vivo* for 3-D data undersampled in two dimensions and with variable density ( $R = 2.63$ ), acquired in a 22-sec breath-hold, reconstructed using generalized SENSE. *In vivo* motion precluded the use of actual multiple replica SNR measurements in this case. [Phantom/abdominal images acquired using 8-channel head/body array-coils. Subject gave written informed consent with IRB approval.]

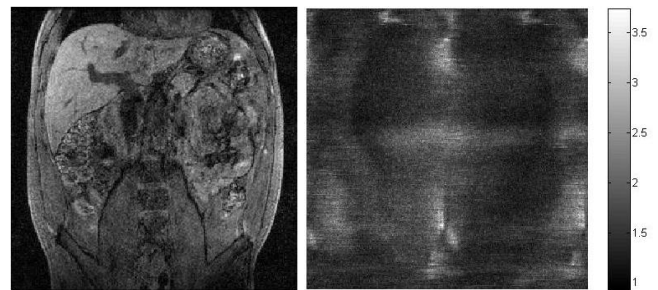


**Figure 1:** Unaccelerated image (top-left) and accelerated image (top-right) for 2-D acceleration ( $R = 3.1$ ) reconstructed with generalized SENSE method; SNR maps from actual multiple image replicas (bottom-left) and the pseudo multiple replica method (bottom-right).



**Figure 2:** SNR maps for the same undersampled data as in Fig.1 reconstructed with a GRAPPA-like method; image noise calculated from stacks of actual (left) and pseudo multiple replicas (right).

**References:** 1. Pruessmann et al. MRM 1999 42:952-62. 2. Kellman et al. MRM 2005 54:1439-47. 3. Pruessmann et al. MRM 2001 46:638-51. 4. Robson et al. MRM (in press) 5. Robson et al. Proc. ISMRM Berlin 2007: 1747.



**Figure 3:** Image (left) from a 3-D volumetric acquisition with acceleration in two dimensions acquired in a 22-sec breath-hold with corresponding  $g$ -factor map (right) calculated using the pseudo multiple replica method.