The GRAPPA Coefficients Estimation using Weighted Least Squares Method

E. G. Kholmovski[1], and S. Vijayakumar[1]
[1]UCAIR, Department of Radiology, University of Utah, Salt Lake City, Utah, United States

Introduction

GRAPPA [1] is an extensively used technique for parallel MRI (P-MRI). The GRAPPA coefficients estimation problem can be considered as a system identification problem where inputs are data acquired by multiple coils, desired outputs are missing k-space data, and system model characteristic for each coil element describes the conversion of input data into outputs. The system model (reconstruction coefficients) can be found by considering the data from the completely sampled central k-space region (autocalibrating data) as known input and output data. This over-determined problem is typically solved using least squares (LS) technique. The LS solution is vulnerable to outliers and low SNR data because each data point contributes equally to the LS solution. It is well know that weighted least squares (WLS) technique produces more accurate solution than that of the LS technique.

Theory and Methods

The GRAPPA coefficient calculation can be described by the following equation: **Ax=b,** where **A** is a matrix with dimensions $L \times Nr$ $(L>>Nr)$, Nr is the number of unknown GRAPPA reconstruction coefficients, L is the number of equations. The LS solution is the solution that satisfies $\min\|\mathbf{Ax}-\mathbf{b}\|_2^2$. $\|\mathbf{Ax}-\mathbf{b}\|_2^2$ can be presented as a sum of two terms. One term is due to noise in measurement data and the second term is due to error between the assumed and the true system models. Thus, non-optimal solution is found when the data are seriously contaminated by noise or when the assumed model does not accurately describe the true system model.

In conventional WLS, weights are chosen such that measurements with low SNR give reduced contributions to the solution. This approach assumes that the problem under consideration has an exact solution and noise may cause deviation from it. In the case of the GRAPPA reconstruction coefficients estimation, the situation is quite different because GRAPPA itself is an approximation to a mathematically accurate formulation of unfolding problem [2]. In GRAPPA, only short range correlations between k-space points in coil datasets are taken into account. Validity of this approximation is determined by coil sensitivity profiles and SNR of k-space points. Error due to exclusion of long range correlations is spatially varying in k-space and depends on energy of k-space points and the level of noise in k-space data. If some k-space point has a high value then its contributions to the neighboring points will be substantial (in comparison with noise level) in a larger neighborhood than if it had a low value. Thus, weights for the WLS method should be chosen such that error due to the GRAPPA approximation is not a dominant term and simultaneously low SNR data do not cause significant deviation from the true solution. To achieve this goal, autocalibrating data can be sorted based on their values. Weights for the WLS solution of the GRAPPA reconstruction coefficients estimation should be assigned such that autocalibrating data with the highest and lowest values give reduced or zero contributions to the solution.

To test the proposed technique, imaging studies were performed on a 3T Trio MR system (Siemens Medical Solutions, Erlangen, Germany) using the eight-channel head coil (MRI Devices, Waukesha, WI). Phantom images were acquired using a GRE pulse sequence with the following imaging parameters: FOV=18x18 cm, 384x384 matrix, 3 mm slice thickness, TR/TE=100/6 ms, a=20^o.

Results

Amplitudes of autocalibrating data are distributed in a wide range as illustrated by the histogram shown in Fig. 1. The peak corresponds to low SNR data, whereas, the tail of the histogram corresponds to high SNR data. Fig. 2 shows the images reconstructed using different sets of reconstruction coefficients. It is obvious that reconstruction coefficients calculated using low SNR or high SNR sets of autocalibrating data is not optimal and produces strongly artifactual images. In case of the low SNR data, noise causes non-accurate solution. Whereas, in the case of high SNR data, GRAPPA approximation (short range correlation) is not completely accurate model. Fig. 3 shows that a small fraction of autocalibrating data with median energy values can give images with better SNR and smaller RMS error in comparison with the images reconstructed using all autocalibrating data. Furthermore, reconstruction with the reduced set of autocalibrating data is more computationally efficient.

Conclusion

WLS approach can be easily utilized to find improved GRAPPA reconstruction coefficients. Weights should be chosen such that contributions to the solution from autocalibrating data with highest and lowest values are reduced or completely excluded.

Acknowledgments

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References: [1] Griswold MA, et al, MRM 2002;47:1202-10. [2] Kholmovski EG, et al, ISMRM 2005, p. 2672.

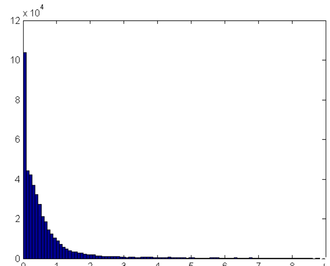

Figure 1. Typical histogram of autocalibrating data energy

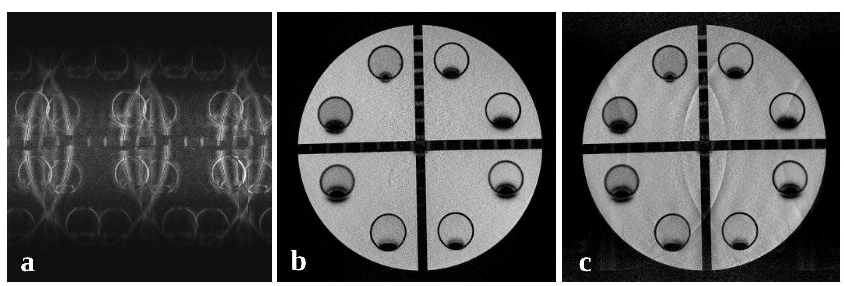

Figure 2. Images reconstructed using the GRAPPA coefficients estimated using 5% of autocalibrating data with: (**a**) lowest signals; (**b**) median signals; (c) highest signals.

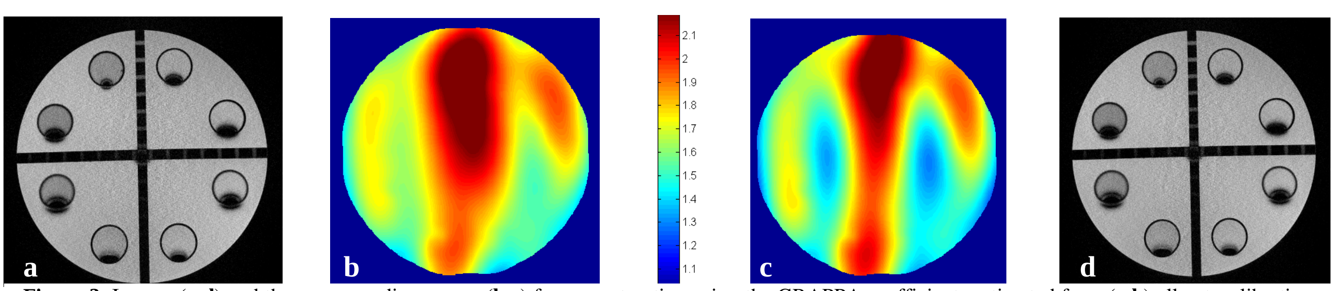

Figure 3. Images (**a,d**) and the corresponding g-map (**b,c**) for reconstruction using the GRAPPA coefficients estimated from (**a,b**) all autocalibrating data (**c,d**) 6.25% autocalibrating data with median energy. For (a): RMS=0.078, recon. time=1.55 sec; for (c): RMS=0.074 and recon time=0.95 sec