G-factor Maps of Conjugate Gradient SENSE Reconstruction

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INTRODUCTION:
In parallel imaging with Cartesian sampling, the spatially varying *g*-factor represents the loss in signal to noise ratio (SNR) due to ill-conditioning of the matrix inverse in SENSE reconstruction, and depends on the acceleration rate, the number of coils, and coil geometry. However, the spatially dependent g-factor of other trajectories (e.g. variable-density or non-Cartesian trajectory) is not well understood. The reconstruction SNR (average over the entire image) has been used to loosely calculate the average g-factor as $SNR_{full}/(\sqrt{R}SNR_{red})$ where R is the acceleration factor. In this abstract, we propose a method to calculate the generalized spatially varying g-factor map for conjugate gradient (CG) SENSE reconstruction with arbitrary trajectories. The method allows us to analyze how different trajectories and number of iterations in CG affect the SNR in a spatially dependent way.

THEORY:
For Cartesian SENSE, the image is reconstructed by solving $\mathbf{v} = (\mathbf{S}^H\mathbf{\Psi}^{-1}\mathbf{S})^{-1}\mathbf{S}^H\mathbf{\Psi}^{-1}\mathbf{a}$ [1] pixel by pixel where v is the desired image vector, $\mathbf{a}$ is the vector of aliased images from all channels, S is the sensitivity matrix (1), Ψ is receiver noise matrix. In this case, the g-factor is defined as $g_\rho = \sqrt{\left[(\mathbf{S}^H\mathbf{\Psi}^{-1}\mathbf{S})^{-1}\right]_{\rho,\rho}(\mathbf{S}^H\mathbf{\Psi}^{-1}\mathbf{S})_{\rho,\rho}}$ [2] at pixel ρ. For arbitrary trajectories, the image can be reconstructed by solving $(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E}\mathbf{v}) = \mathbf{E}^H\mathbf{m}$ [3] (**m** is sampled *k*-space data, $\mathbf{E}$ is the encoding matrix as in (1,2)) iteratively using CG method to approximate $\mathbf{v} = (\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})^{-1}\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{m}$ [4] numerically. In this case, the g-facor is given by $g_\rho = \sqrt{\left[(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})^{-1}\right]_{\rho,\rho}(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})_{\rho,\rho}}$ [5]. The same iterative CG method can be used to calculate the first term $\left[(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})^{-1}\right]_{\rho,\rho}$ in Eq. [5]. Specially, we calculate $(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})\tilde{\mathbf{v}} = \mathbf{b}$ using the the iterative CG method, where $\mathbf{b}$ denotes an all-zero image except at pixel ρ whose value is unit one. After several iterations, the value of the obtained "image" $\tilde{\mathbf{v}}$ at the corresponsing pixel ρ gives the approximation of $\left[(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})^{-1}\right]_{\rho,\rho}$. The second term in Eq. [5] $(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})_{\rho,\rho}$ does not need matrix inversion and can be easily obtained by taking the pixel ρ of the image obtained by a forward encoding $(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})\mathbf{b}$.

METHOD AND RESULTS:
We acquired a water phantom data on a Hitachi Airis Elite (Kashiwa, Chiba, Japap) 0.3T permanent magnet scanner with a four-channel head coil and a single slice spin echo sequence (TE/TR = 40/1000ms, 8.4KHZ bw, 256*256 matrix size, FOV = 220 mm^2). The sensitivity maps were estimated using the full k-space data. We compared the g-factors at a reduction factor of 4 for three cases: (a) basic SENSE (using matrix inversion) with uniform Cartesian trajectory; (b) CG SENSE with uniform Cartesian trajectory; and (c) CG SENSE with variable-density (VD) Cartesian trajectory (32 fully sampled central lines and reduction factor of 4 outside). We also compared the g-factors after 3 and 8 CG iterations. The results are shown in Figure 1.

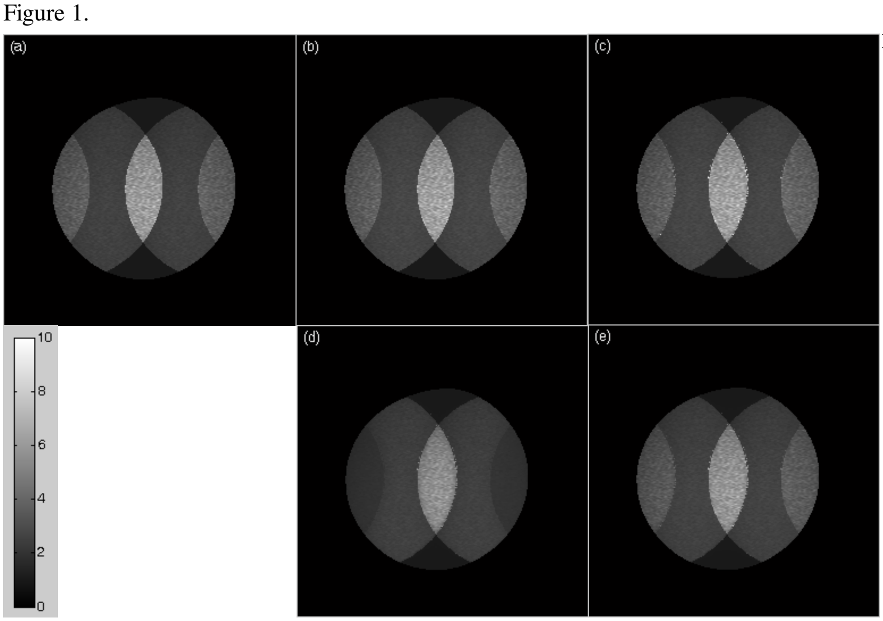

Fig 1: G-factor maps for R= 4 using (a) basic SENSE, (b) CG with 3 iterations, (c) CG with 8 iterations, (d) CG method from VD data with 3 iterations, and (e) CG method from VD data with 8 iterations. All results are scaled to the same range.

DISCUSSION:
Our results show that the g-factor of the CG SENSE reconstruction has similar spatial variation pattern as that of the basic SENSE reconstruction. However, the value of g-factor in CG SENSE depends on and increases with the number of iterations. It explains the semi-converge property of CG SENSE (3): increasing iterations reduces the aliasing artifacts but increases the noise at the same time, which can be observed in VD-CG case. Proper stopping criterion should be used to balance the aliasing artifacts and noise. In addition, our results show the VD trajectory improves the g-factor at small number of iterations, but does not improve much as iteration number increases. The proposed method can be used to calculate the g-factor for spiral and radial trajectories, as well as to evaluate the SNR improvement by the regularization technique for non-Cartesian SENSE (4).

REFERENCES:
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