Parallel Image Reconstruction using a Single Signal in Phase-Scrambling Fourier Imaging Technique

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INTRODUCTION: A novel image reconstruction technique is proposed in which parallel image reconstruction is performed based on the SENSE algorithm using only a single set of signals. The signal obtained in the phase-scrambling Fourier transform (PSFT) imaging technique can be transformed to the signal described by the Fresnel transform of the objects, which is known as the equation of diffracted wave-front equation of the object in acoustics or optics. The application of a weighting function to the PSFT signals has a similar effect as the application of a sensitivity function to the object function itself. Therefore, we can obtain two or more folded images from a single set of signals, and image reconstruction based on the SENSE algorithm is possible using a series of folded images given different weighting functions.

METHOD: Phase-scrambling Fourier transform (PSFT) imaging[2] is a technique whereby a quadratic field gradient is added to the pulse sequence of conventional FT imaging in synchronization with the field gradient for phase encoding. The signal obtained in PSFT is given by Eq. (1):

$$v(k_x,k_y)=\int\int_{-\infty}^{\infty}\rho(x,y)e^{-j\gamma b\tau(x^2+y^2)}e^{-j(k_x x+k_y y)}dxdy, \quad (1) \qquad v(x',y')e^{-j\gamma b\tau(x'^2+y'^2)}=\int\int_{-\infty}^{\infty}\rho(x,y)e^{-j\gamma b\tau[(x'-x)^2+(y'-y)^2]}dxdy. \quad (2)$$

where $\rho(x,y)$ represents the spin density distribution in the subject, γ is the gyromagnetic ratio, and b and τ are the coefficient and impressing time, respectively, of the quadratic field gradient. Equation (1) can be rewritten as the Fresnel transform equation, as shown in Eq. (2), by using the variable substitutions $x' = -k_x/2\gamma b\tau$ and $y' = -k_y/2\gamma b\tau$ [3]. Subject images can be obtained by applying the inverse Fresnel transform from the signal written as Eq. (2) which is a convolution integral of $\rho(x,y)$ and $\exp[-j\gamma b\tau(x^2+y^2)]$.

The Fresnel transformed signal with the sensitivity of detector coil $S(x,y)$ transformed from the PSFT is written as the middle part of Eq.(3). When the width of $F[S(x,y)]$ is very small compared to $F[\exp[-j\gamma b\tau(x^2+y^2)]]$, the middle part of Eq.(3) is approximated by right-hand part of Eq.(3) and that equation can be transformed to right-hand part of Eq.(4). Equations (3) and (4) show that application of a sensitivity functions to the Fresnel transformed signal has a similar effect as the application of a sensitivity function to the object function $\rho(x,y)$.The amplitude of Fresnel transformed signal and PSFT signal are equal, as shown in Eq.[2], so the relation of Eq.[4] holds to the PSFT signal, too.

$$[S(x,y)\rho(x,y)]*e^{-j\gamma b\tau(x^2+y^2)} \overset{\text{FT}}{\Rightarrow} (F[S(x,y)]*F[\rho(x,y)])\cdot F[e^{-j\gamma b\tau(x^2+y^2)}] \approx F[S(x,y)]*(F[\rho(x,y)]\cdot F[e^{-j\gamma b\tau(x^2+y^2)}]) \quad (3)$$

$$F[S(x,y)]*(F[\rho(x,y)]\cdot F[e^{-j\gamma b\tau(x^2+y^2)}]) \overset{\text{IFT}}{\Rightarrow} S(x,y)\cdot(\rho(x,y)*e^{-j\gamma b\tau(x^2+y^2)}) = S(x,y)\cdot v(x',y') \quad (4)$$

Figure 1 shows a schematic of the proposed method. Fig.1(a) shows the amplitude of PSFT signal. It was shown that the amplitude distribution in *k*-space is similar to the blurred image of the object. Fig1(c) shows the PSFT signal multiplied by the dummy sensitivity map shown in part (b), which was calculated numerically. Fig.1 parts (d) and (e) are the folded images reconstructed by (a) and (c), respectively. Application of the SENSE algorithm using folded images (d) and (e) with sensitivity map (b) gives an unfolded image (f).

EXPERIMENTS: Fig.1 shows the simulation experiment corresponding to SENSE factor 2. The equivalent sampling step in the Fresnel-transformed signal domain $\Delta x'$ is set to $2\Delta x$ so that the equivalent sampling width in Eq.(2) is equal to the field-of-view of unfolded image. A fairly good image was obtained, as shown in Fig.2(b). Our results using a low-field MRI data show that the proposed method can be applied to noisy signal data or to an object with phase variations.

CONCLUSION: A new unfolding image reconstruction algorithm that uses only a single signal from the phase-scrambling Fourier imaging technique is proposed. Even though the error of the unfolding calculation increases as the number of SENCE factors increases, the proposed method has the advantages of a homogeneous signal-to-noise ratio in the image and no error in the sensitivity function since it is given numerically in the reconstruction algorithm. This method could be applied to commercial MRI simply adding a weak quadratic field gradient to the imaging pulse sequences. In the future, we will investigate the improvement of images by correcting the sensitivity map using the previously obtained image.

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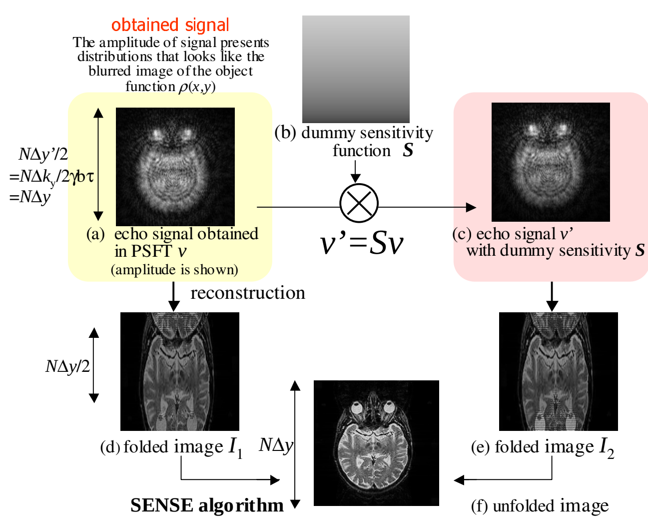

←**Fig.1** Schematic of parallel imaging using a single signal; dummy sensitivity map(b) is multiplied to PSFT signal (a) to obtain a PSFT signal having sensitivity close to *S*.

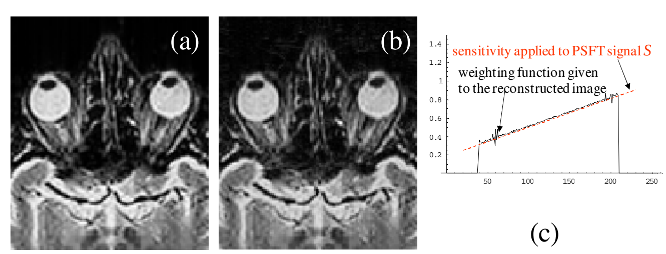

Fig.2 Simulation results of the proposed method; (a) fully sampled image, (b) proposed method with 2-fold under-sampled data, (c) the weight function applied to image compared to dummy sensitivity function applied to the PSFT signal