Non-Cartesian Parallel Reconstruction Using Null Operations (NC-PRUNO)

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INTRODUCTION Non-Cartesian parallel imaging has been widely used since the innovation of conjugate gradient SENSE (CG-SENSE) reconstruction algorithm (1). Recently, several k-space reconstruction methods have also been developed for non-Cartesian trajectories (2-4). In most of these methods, each Cartesian k-space sample is synthesized one by one by linearly combining some acquired samples. Here we present a novel iterative reconstruction approach, in which we compute all missing interleaved non-Cartesian samples while assuring the consistency of all acquired samples at the same time. This method is based on our recently developed method for Parallel Reconstruction Using Null Operations (PRUNO) that has been applied for Cartesian trajectory. Here we demonstrate an algorithm for non-Cartesian Parallel Reconstruction Using Null Operations (NC-PRUNO). Although we demonstrate NC-PRUNO here by using variable density spiral (VDS) trajectories, it can also be applied to any other interleaved trajectory such as radial when data calibration is available.

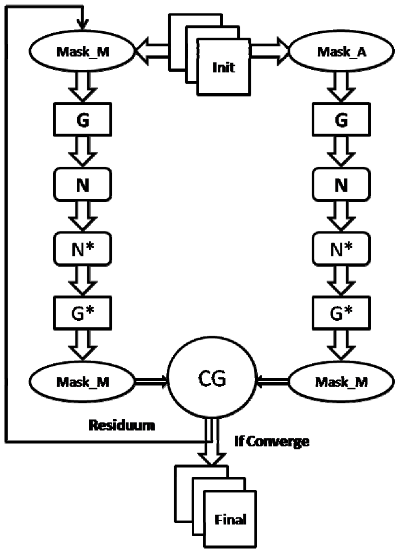

Figure 1: The iterative algorithm for NC-PRUNO reconstruction. Here N and N^* represent null operation and its conjugate respectively. G is gridding and G^* is inverse-gridding. *Mask_M* and *Mask_A* refer to masking out only missing samples or only acquired samples respectively.

METHODS In parallel imaging, due to the smoothness of coil sensitivity profiles, nearby k-space samples from multi-channels are highly correlated. Approximately, a certain subset of local neighbors may exhibit a linear dependence that is also shift invariant. Thus, there exists a non-zero linear operator, which nulls the corresponding subset of local samples. As in other auto-calibration based methods, we assume that a fully sampled k-space region is available. Then by choosing different neighbor templates, we can obtain multiple null operators from data calibration. In PRUNO, by using multiple null operators, the image reconstruction can be formulated as an overdetermined linear equation $Nx = 0$. [1]

Here N is a sparse encoding matrix which concatenates all null operators and x is the vectorized desired full-grid multi-coil k-space data. There are many options on how to choose a set of neighbor templates. One good choice is to simply use a set of GRAPPA operators (7) by setting the coefficient of each target sample as -1. If we approximate the relationship between Cartesian samples and non-Cartesian samples using gridding, we can obtain $NGd = 0$ [2], where G is another sparse encoding matrix corresponding to gridding and d is a vector concatenated with "full" interleaved non-Cartesian samples. By doing proper permutation, we can decompose Eq [2] as

$$N\begin{bmatrix} G_m & G_a \end{bmatrix}\begin{bmatrix} d_m \\ d_a \end{bmatrix} = 0 \quad [3]$$

Here the two subscripts m and a represent missing and acquired, respectively. Obviously, the goal of the reconstruction is to solve d_m, and the final system equation turns to be

$$(G_m^* N^* N G_m) d_m = -(G_m^* N^* N G_a) d_a \quad [4]$$

Since our null operators are simply composed of convolutions and additions, this equation can be solved effectively by using a conjugate gradient method, which is similar to a CG- SENSE reconstruction. An iterative reconstruction scheme is shown in Fig 1. After solving the system, an image for each coil can be reconstructed by doing gridding and the final image can be obtained by using sum-of-squares.

RESULTS Our method has been applied to both in-vivo and simulated data. We used 8 coil channels for both experiments. VDS trajectories were used here with a reduction factor of two. Fig 2 compares the sum-of-square reconstructions between direct regridding and NC-PRUNO. Fig 3 shows the CG iterating progress for a NC-PRUNO reconstruction.

DISCUSSION AND CONCLUSION Here we have demonstrated a k-space iterative non-Cartesian parallel imaging method. The reconstruction is efficient since we don't need to do multiple data calibrations for every Cartesian sample or for every block of samples. The algorithm is also very easy to implement. Another advantage of NC-PRUNO is that it truly maintains the consistency of all acquired samples. Due to the imperfection of gridding operations, the convergence of the algorithm can be slow or unstable as the image size gets larger. Proper regularization may be helpful to improve the performance and we are still investigating this problem.

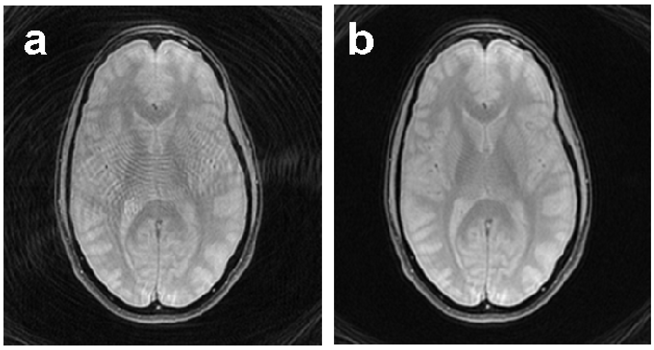

Figure 2: Reconstructed sum-of-square images. (a) shows the image reconstructed from direct regridding; (b) shows the image reconstructed from NC-PRUNO.

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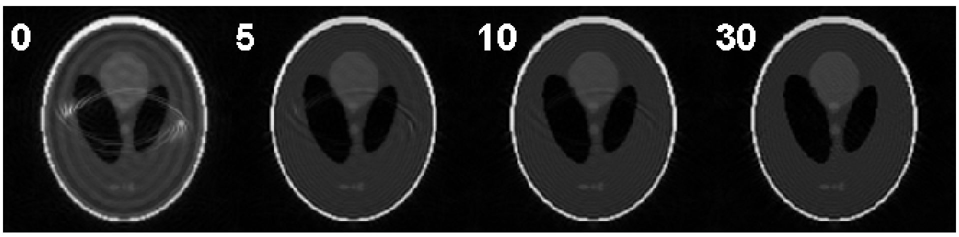

Figure 3: NC-PRUNO reconstruction results at different iterations.

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