A Variable Projection Method to JSENSE

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INTRODUCTION

In parallel imaging, SENSE requires accurate knowledge of coil sensitivities for a faithful reconstruction (1). The self-calibrating technique (2) for sensitivity estimation has been well accepted, especially for dynamic imaging, to eliminate the need for a separate calibration scan, thus avoiding misregistration artifacts. JSENSE (3) has recently been proposed to improve the accuracy of sensitivity estimation using the self calibration data. It regards both the coil sensitivities and the desired images as unknowns to be solved for jointly and formulates the reconstruction as a nonlinear problem. The existing algorithm solves the problem by iterative alternating minimization, which requires considerable number of self calibration data for an accurate initial sensitivity estimation. In this abstract, we propose to use the variable projection method (4) to solve the nonlinear optimization problem. This method requires very few self calibration data because it converges to an optimal solution regardless of the initial value. The proposed method has been tested on a set of simulation data and demonstrated promising results.

THEORY

In JSENSE, the problem is formulated as $\mathbf{E}(\mathbf{a})\mathbf{f} = \mathbf{d}$ [1], where $\mathbf{E}(\mathbf{a})$ is the sensitivity encoding matrix with $\mathbf{a}$ being the unknown parameters for coil sensitivities (e.g. polynomial coefficients as in (3)), $\mathbf{f}$ is the image to be reconstructed, and $\mathbf{d}$ is the acquired k-space data.. In present of data noise, both unknowns $\mathbf{a}$ and $\mathbf{f}$ can be obtained simultaneously by the least squares solution $\{\mathbf{a},\mathbf{f}\} = \arg\min_{\mathbf{a},\mathbf{f}} \|\mathbf{d} - \mathbf{E}(\mathbf{a})\mathbf{f}\|_2$ [2]. To efficiently find the optimal solution to [2], we use the variable projection (VP) method (4). Specifically, the SENSE solution $\mathbf{f} = [\mathbf{E}^H(\mathbf{a})\mathbf{E}(\mathbf{a})]^{-1}\mathbf{E}^H(\mathbf{a})\mathbf{d}$ [3] is plugged into Eq. [2], and solution to $\mathbf{a} = \arg\min_{\mathbf{a}} \left\|\mathbf{d} - \mathbf{E}(\mathbf{a})[\mathbf{E}^H(\mathbf{a})\mathbf{E}(\mathbf{a})]^{-1}\mathbf{E}^H(\mathbf{a})\mathbf{d}\right\|_2$ [4] gives the sensitivity parameter $\mathbf{a}$. The desired image $\mathbf{f}$ can be finally reconstructed by Eq. [4]. It was proved that the variable projection solution is the same as the solution to the original problem in [2] (4). Details of the variable projection algorithm are given on the right.

<u>Variable Projection Algorithm</u>:

Initialize $\mathbf{a}^0$;

For k=0, 1, …

Compute $\mathbf{E}(\mathbf{a}^k)$, $\mathbf{DE}(\mathbf{a})$ where $\mathbf{D}$ is the derivative operator;

Perform QR decomposition: $\mathbf{E(a)=QR}$; Q is an orthogonal matrix, and $\mathbf{R} = \mathbf{Q}^H\mathbf{E}(\mathbf{a}^k) = \begin{bmatrix} \mathbf{T_{11}} & \mathbf{T_{12}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, where $\mathbf{T_{11}}$ is a square upper triangular matrix of r_0=rank(E);

$$\mathbf{P}_{\mathbf{E}(\mathbf{a}^k)} = \mathbf{Q}^H \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{\mathbf{m}-\mathbf{r_0}} \end{bmatrix} \mathbf{Q};$$

$$\mathbf{E}_B(\mathbf{a}^k) = \begin{bmatrix} \mathbf{T}_{11}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{Q};$$

$$r(\mathbf{a}^k) = \mathbf{P}_{\mathbf{E}(\mathbf{a}^k)}\mathbf{d};$$

$$\mathbf{D}r(\mathbf{a}^k) = -[\mathbf{P}_{\mathbf{E}(\mathbf{a}^k)}\mathbf{DE}(\mathbf{a}^k)]\,\mathbf{E}_B(\mathbf{a}^k)\mathbf{d} - \mathbf{E}_B^H(\mathbf{a}^k)[\mathbf{P}_{\mathbf{E}(\mathbf{a}^k)}\mathbf{DE}(\mathbf{a}^k)]^H\mathbf{d}$$

$$\mathbf{a}^{k+1} = \mathbf{a}^k - t^k[\mathbf{D}r(\mathbf{a}^k)]^+\,\mathbf{P}_{\mathbf{E}(\mathbf{a}^k)}\mathbf{d};$$

end

Reconstruct image $\mathbf{f} = [\mathbf{E}^H(\mathbf{a})\mathbf{E}(\mathbf{a})]^{-1}\mathbf{E}^H(\mathbf{a})\mathbf{d}$

METHOD AND RESULTS

The proposed approach was tested on a set of simulated data where both the phantom and sensitivities were obtained from real scans. A four channel coil was used. The data were simulated to achieve a reduction factor of 2 with 8 self calibration data lines. The performance of the proposed algorithm can be evaluated visually in Fig. 1. It is seen that the proposed variable projection (VP) method greatly reduces the image aliasing artifacts in the SENSE reconstruction using the conjugate gradient (CG) method (5) with self-calibrated sensitivities.

DISCUSSION

The variable projection method significantly reduces the number of self calibration data needed during the accelerated scans, and thus reduces scan time. The method has the disadvantage that it requires explicit expression of the encoding matrix, which is memory demanding and computational intensive. Efficient implementation of the method will be investigated in future study.

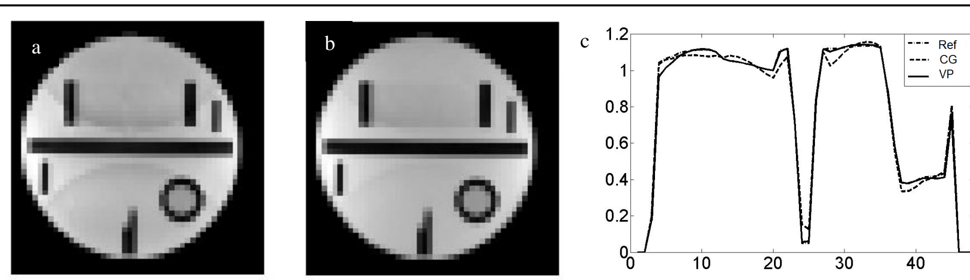

Fig. 1 Simulation results (a) CG SENSE reconstruction using self-calibrated sensitivities (b) VP reconstruction (c) comparison of CG and VP vertical profiles with the original image

REFERENCES

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