Time-Domain Finite-Difference/Finite-Element Hybrid Simulations of High-Field RF Coils

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Introduction: Full-wave electromagnetic simulations that rigorously solve Maxwell's equations are important in high-field RF coil design. Although the popular FDTD method is good at modeling the inhomogeneous human body, it is inadequate for modeling curved coil structures (1). Methods that accurately model curved coil structures, such as the Finite-Element method (FEM) and the surface integral-equation (SIE) method (2, 3), are computationally expensive in terms of modeling inhomogeneous human body. Here we present a stable Time-Domain Finite-Difference/Finite-Element (TD-FD/FE) method that hybridizes the FDTD and the FEM. The FEM is only applied to model curved coil structures while the human body is simulated by the FDTD. Since both methods are applied simultaneously in one simulation, the interaction between the human body and RF coils is fully accounted for with well balanced numerical accuracy and computational efficiency.

Methods: The TD-FD/FE hybrid method utilizes unstructured grids in the FEM region while structured Cartesian grids in the FDTD region (Yee's scheme). In the FEM region, the two *curl* equations are combined into the so-called *curl-curl* equation

$$\nabla \times \frac{1}{\mu}\nabla \times \vec{E} + \sigma\frac{\partial \vec{E}}{\partial t} + \epsilon\frac{\partial^2 \vec{E}}{\partial t^2} + \frac{\partial \vec{J}}{\partial t} = 0$$

The electric fields in the *curl-curl* equation are firstly discretized by a set of *curl*-conforming basis functions. Then the Galerkin method is applied to establish a linear system of equations. Finally, the time-domain FEM is obtained by using the Newmark Beta method

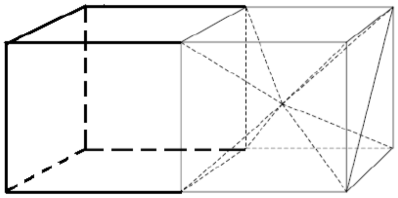

Fig. 1: The unstructured/Cartesian grids interface.

$$\frac{1}{\mu}[S]_{ji}\left[\beta e_i^{n+1} + (1-2\beta)e_i^{n} + \beta e_i^{n-1}\right] + [T]_{ji}\left[\sigma\frac{(e_i^{n+1}-e_i^{n-1})}{2\Delta t} + \epsilon\frac{(e_i^{n+1}-2e_i^{n}+e_i^{n-1})}{\Delta t^2}\right] = 0$$

where *[S]* and *[T]* represents stiffness and mass matrix respectively. It has been proven that when β>= ¼, the time-domain FEM is unconditionally stable regardless of the size of individual elements. Furthermore, the FDTD method is a special FEM scheme by applying β=0 and integral-lumping inside each Cartesian element. Thus the FDTD and the time-domain FEM methods can be naturally hybridized. In practice, tetrahedral elements are preferred to model arbitrarily shaped geometries in three-dimension. In order to connect them to Cartesian grids, a layer of pyramidal elements can be applied (Fig. 1).

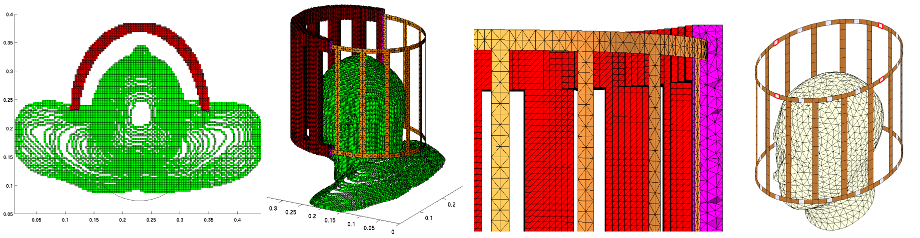

Fig. 2: The hybrid grids of the coil and the human body (left-most and mid-left), detailed view of the tetrahedral mesh (mid-right) and the SIE model of the same coil (right-most). Note that the shield is not shown.

Results and Discussion: A C++ program was developed and used to simulated the B_1 and electric field distributions of a 7.0 Tesla 16-rung shielded elliptical high-pass birdcage coil (12-in by 8.9-in axial ratio, 10-in high). The shield is 1-in away from the coil. The results were compared with those of the SIE method (3). In the SIE simulation, the Specific Anthropomorphic Mannequin (SAM) model filled with brain equivalent (ε_r=52 and σ=0.552) was used (Fig. 2). In the TD-FD/FE hybrid simulation, a 3x3x3 mm^3 realistic human model (NLM's Visual Man Project) was used. The *exact* geometry of the birdcage coil was modeled with tetrahedral grids (Fig. 2). The overall number of elements in the unstructured FEM region was about 400,000. The maximum and the minimum dihedral angles of the tetrahedral grids were 147° and 14° respectively. The resulting FEM matrix contained about 1 million unknowns and it was decomposed by an in-house developed sparse Cholesky solver, which took 180 seconds on a 2.6 GHz CPU. Inversion of the FEM matrix during each time step was accomplished by back substitution. A first-order derivative Gaussian pulse was used for 4-port quadrature drive of the coil. Results were obtained after 30,000 time steps. The coil was numerically tuned with thirty-two 5.92-pF capacitors by using the SIE method and *exactly* the same capacitance was applied in the TD-FD/FE hybrid simulations. The simulated B_1 and electric field distributions on corresponding slices are compared in Figs. 3 and 4 respectively. In each individual figure, the field is normalized to its own peak. It is observed that by modeling coil geometry exactly and using the same capacitance, the coil is resonant at 298 MHz in both simulations. The B_1 distributions resemble reasonably well despite the differences between the human models, though the SIE results look more inhomogeneous. However, the electrical field distribution exhibits quite large differences because the SIE method failed to resolve local hot spots due to the lack of anatomical detail. These local electrical field hot spots are critical in evaluating RF safety issues.

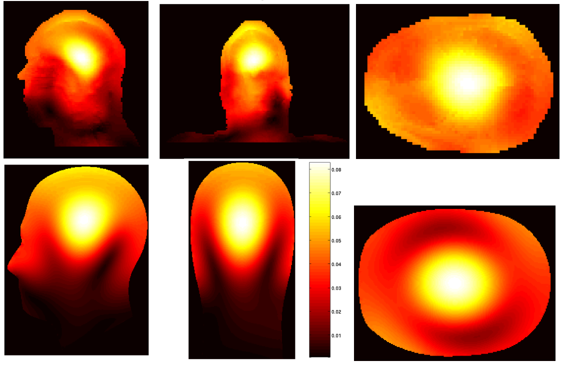

Fig 3: B_1 maps calculated by the TD-FD/FE hybrid method (top row) and the SIE method (bottom row) on corresponding slices.

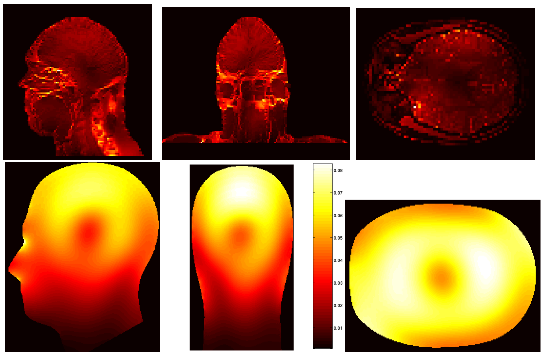

Fig 4: Electric field maps calculated by the TD-FD/FE hybrid method (top row) and the SIE method (bottom row).

Conclusion: We presented a TD-FD/FE hybrid approach that simultaneous applies the FDTD and the FEM methods in a single simulation. This method is accurate and efficient in simulating the interaction between the human body and RF coils at high-field.

References: 1)Yee KS, IEEE TAP 14:302-307(1966) 2) Petropoulos LS et. al, MRM 30:366-372(1993) 3) Wang S et. al, PMB 51:3211-3229(2006)