A BiCG Solution based Quasi-Static Finite-Difference Scheme for Induced Field Evaluation in MRI

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Synopsis: Magnetic resonance technology employs a wide range of electromagnetic frequencies and field strength for rapid generation of high-resolution anatomical images. These electromagnetic fields are known to interact with the living tissue in a number of different ways. This study presents a biconjugate gradient method (BiCG) that can significantly improve the performance of the quasi-static finite-difference scheme (QSFD), which has been widely used to model field induction phenomena in voxel phantoms. The wide capability and superior computational performance of the BiCG method is demonstrated by modelling the exposures of MRI healthcare workers to fields produced by pulsed field gradients, which is presently an important topic of research in light of the Physical Agents Directive (PAD) 2004/40/EC. A variety of realistic operator postures near the bore entrance of an MRI system are modeled.

Introduction: The conventional QSFD algorithm solves an implicit equation system by employing the successive over-relaxation (SOR) algorithm to iteratively search for the solution. When the problems to be analyzed are of large scale, such as in instances of high resolution inhomogeneous phantoms or spatially non-uniform source field distributions, the system of equations can become significantly ill-conditioned. In that case, the conventional QSFD-SOR method suffers from poor numerical convergence, long computing times and large memory costs, while in some situations the solution will not converge at all. Considering that only a relatively small number of the matrix elements are nonzero, the developed formulation exhibits a band diagonal sparse matrix of an asymmetric form, which can be solved efficiently and effectively using the BiCG technique.

Method: The total electric field inside the body model can be split into primary field $\vec{E}_1$ and secondary field $\vec{E}_2$, according to:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = -\frac{\partial \vec{A}}{\partial t} - \nabla\Phi$$

Here, $\vec{A}$ denotes the vector magnetic potential due to the source and Φ is the scalar electric potential.

Based on QSFD [1], the computation of the electric fields is given by the governing surface integral equation:

$$\int_s (\sigma \frac{\partial \vec{A}}{\partial t}) \cdot dS = -\int_s (\sigma \nabla\Phi) \cdot dS$$

where S represents the surface of the integral region and σ is the sample conductivity.

The integral equation is then expressed in discrete form:

$$\sum_{q=0}^{1} (\sigma^a_{i+q\times2-1,j,k} + \sigma^a_{i,j+q\times2-1,k} + \sigma^a_{i,j,k+q\times2-1}) \cdot \Phi_{i,j,k} - \sum_{q=0}^{1} \sigma^a_{i+q\times2-1,j,k} \cdot \Phi_{i+q\times2-1,j,k}$$
$$- \sum_{q=0}^{1} \sigma^a_{i,j+q\times2-1,k} \cdot \Phi_{i,j+q\times2-1,k} - \sum_{q=0}^{1} \sigma^a_{i,j,k+q\times2-1} \cdot \Phi_{i,j,k+q\times2-1}) = -f(\vec{A})h$$

Where:

$$f(\vec{A}) = \sum_{q=0}^{1} ((\sigma^a_{(i+q\times2-1,j,k)}\, \partial\vec{A}/\partial t_{(i+q\times2-1,j,k)}) \cdot s_x^q +$$
$$(\sigma^a_{(i,j+q\times2-1,k)}\, \partial\vec{A}/\partial t_{(i,j+q\times2-1,k)}) \cdot s_y^q + (\sigma^a_{(i,j,k+q\times2-1)}\, \partial\vec{A}/\partial t_{(i,j,k+q\times2-1)}) \cdot s_z^q)$$

then be expressed as a linear relation in the matrix form of $A \cdot x = b$, which appears to be a sparse matrix form and can be solved efficiently by BiCG method.

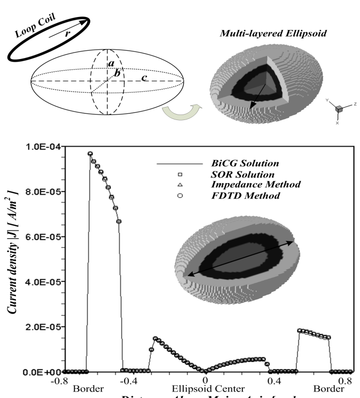

Fig.1 Validation involving a multi-layered ellipsoid excited by a current carrying loop at an angle. The induced current density results obtained with the proposed QSFD-BiCG scheme are successfully compared against other known solutions such as QSFD-SOR, finite-difference time-domain (FDTD) and impedance method.

The proposed QSFD-BiCG scheme has been validated against other computational methods (Fig. 1). To demonstrate the computational performance of the proposed method, we have evaluated exposures of MRI occupational workers to fields produced by switched gradient coils in many clinically feasible body postures near the bore entrance of model MRI systems (Fig. 2-4). It is hoped that these particular examples will promote the potential of the QSFD-BiCG method for efficient numerical modelling of worker exposures in MRI and related settings. It is important to mention here that the QSFD-SOR method failed to find a solution in this application as the results did not converge. Compared to the standard QSFD-SOR algorithm, the proposed QSFD-BiCG method offers notable advantages in terms of guaranteed and improved solution convergence, smaller memory burden and the flexibility to handle solution errors, as shown in the table.

Performance results

Property of Coefficient Matrix	Problem Resolution			
	8MM	6MM	4mm	2mm
Scale (n)	1.5e5	3.5e5	1.1e6	8.6e6
Nonzero elements	8.8e5	2.1e6	7.1e6	5.7e7
	Convergence performance			
Parallelized SOR	2.5mins	14mins	36mins	31hours
BiCG	10secs	39secs	3mins	1.8hours
Parallelized BiCG	7secs	23secs	1.5mins	50mins

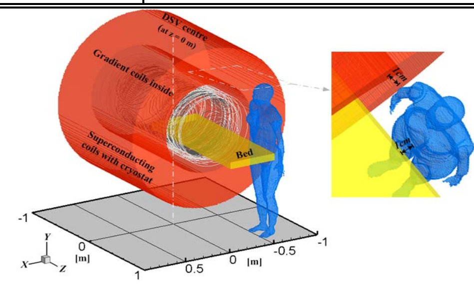

Fig.2 Sketch of the typical posture of an MRI operator at the side of the patient table (bed) near the imager end. The bed is assumed to be 0.4m in width. The distance between the patient and bed is 1cm as indicated.

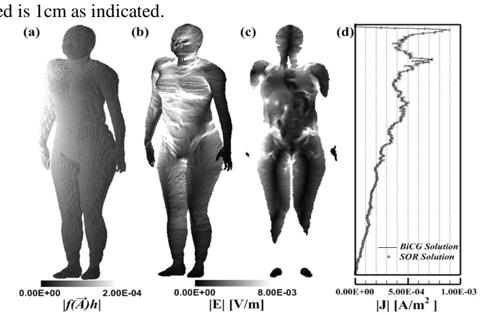

Fig.3 Induced spatial field distributions on the surface and inside the female voxel phantom: **(a)** Vector magnetic potential, **(b)** electric field and **(c)** average current density versus body height. Subplot (c) compares the results between the QSFD-BiCG and QSFD-SOR methods with less than 1% of relative deviation.

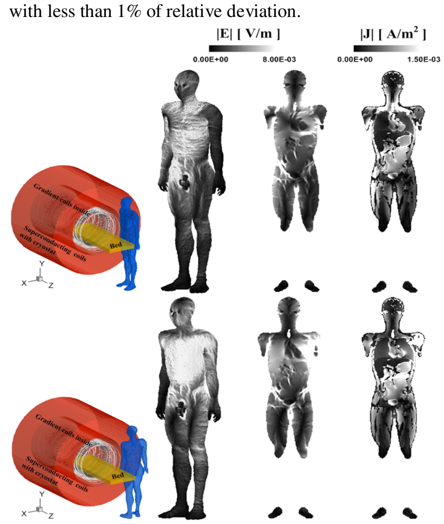

Fig.4 Surface and coronal / sagittal plots of electric field and current densities in the male voxel phantom induced during exposure to the combination of all three gradient coils. Left sketches show the corresponding body postures near the imager end/ patient table.

Conclusion: The results demonstrate that the QSFD-BiCG method provides more robust solutions than the conventional QSFD-SOR scheme can offer. The QSFD-BiCG method can be applied to a variety of MRI-related large-scale electromagnetic field (EM) problems.

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References: F. Liu *et. al.*, *IEEE Trans. Biomed. Eng.*, 50 (7): 804-815, 2003.