Exchange of surface coil antenna patterns due to gyrotropism

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Introduction- Gyrotropic (or "non-reciprocal") media, such as those containing nuclear spins, are characterized by their response to oscillatory fields applied in the presence of a steady magnetic field. In NMR, a linearly polarized RF field applied normal to the main field gives a circularly polarized response whose sense reverses if the direction of the main field is reversed. This behavior is characteristic of gyrotropic media, in which the consequence of steady field reversal is transposition of the (asymmetric) susceptibility tensor. Harrington and Villenueve formulated a reciprocity principle valid for gyrotropic media [1], which has recently formed the basis for a new treatment of reciprocity in NMR transduction [2,3]. The key evidence for the new theory is the near perfect exchange of the transmit and receive patterns of a pair of bilaterally symmetric surface coils – operating as a T/R pair -- when the roles of transmitter and receiver are swapped, forming strikingly different images in the two cases. In the present study, we investigate further predictions of the new theory, particularly regarding Harrington's statement that "the transmitting pattern of any antenna containing gyrotropic media is the same as the receiving pattern with the dc magnetic field reversed". We present direct verification of this statement through experiments on two oppositely polarized 3.0T scanners.

Theory and Results- MR images may be written as the product of coil transmit and receive patterns [3]. The transmit pattern is proportional to the complex magnetization created per unit current in the coil, while the receive pattern is proportional to the complex voltage [3,4] induced in the coil per unit complex magnetization at some point in space. These transmit and receive patterns are formed mathematically by pre- and post-multiplication, respectively, of the spatially dependent coil transmit fields with the RF susceptibility tensor (cf Eq. 1). For the orientation of the tensor shown below, the transmit pattern is proportional to $H_x + iH_y$ and the receive pattern to $H_x - iH_y$. Upon main field reversal, the susceptibility tensor is transposed, which swaps the patterns. For a pair of *receive-only* surface coils with bilateral symmetry, identical patterns are expected; but residual reactive coupling (by the coil admittance matrix, cf Eq. 2) causes a differential artifact, with each coil experiencing a distinct and different pattern of signal loss. These receive patterns (or images, assuming uniform transmission) are predicted to swap from one coil to its mate upon main field reversal. This is shown in the first experiment (cf Fig. 1). Gradient echo images of a cylindrical phantom (cf Methodology below) were collected from a bilaterally symmetric receive array of two elements [5], on oppositely polarized 3.0T scanners – Scanner I and Scanner II. The left coil in Scanner I shows a crescent (or divot) of lost signal in its upper left quadrant; for Scanner II this pattern is swapped, and appears in the right coil.

On the other hand, for the same array configured as a transmit-receive pair (i.e. blocking diodes [4] removed), the gyrotropic model predicts [3] that the transmit or receive pattern for each element is exchanged with the contralateral element's opposite pattern (with orientation reversed) upon either transmit / receive role reversal (as previously shown) or main magnetic field reversal. *The transmit and receive patterns for both array elements can thus be inferred from the results of the receive-only experiment.* This fact was used to predict [3] the distinctly differing images obained by swapping the transmit and receive ports of the array for a fixed direction of the main field. The different images are seen, for example, in the top left and right images of Fig. 2 (Scanner I, left image transmit left port, right image transmit at right port.). A further consequence is that the roles of transmit and receive should swap with reversal of the main field. This is shown in bottom images of Fig. 2: Scanner II left, transmit left port, right, transmit right port. Note that the transmitter ports are always identifiable by a slight shading of intensity, due to partial saturation of the spins. The images in all cases closely resemble the appropriate products of the antenna patterns (properly exchanged and/or oriented) taken from the results of the first experiment (Fig. 1). For example, the mushroom shape is generated on one scanner when transmitting on the left, as the product of flipped bottom right receive pattern (left element transmit pattern) and the flipped top left receive pattern (right element receive pattern). As noted, at the given TR and flip angle, the images are affected by saturation on the transmit side.

Discussion- The gyrotropic model correctly predicts the swappping, due to field reversal, of transmit and receive antenna patterns. The full theory [3] makes to no reference to rotating frame transformations, relying instead on pre and post multiplication field vectors with the susceptibility tensor. A small dielectric effect is also present, but not of fundamental importance [3].

Methodological Details- The dual octagonal array used for the experiments measured 7.4cm wide by 8.4cm long, and was overlapped to minimize mutual inductance. It had a measured crosstalk |S21| = -15.0dB when loaded with the imaging phantom (.025M NaCl), a cylindrical jug of radius 9.25cm. Vitamin capsules were affixed to the coil former to mark the positions of the coil elements in the images. Scans were conducted on two GE 3.0T scanners (GE Healthcare, Waukesha, WI), with opposite main field polarity, as verified from installation records. All scans were single slice spoiled gradient echo (SPGR) acquisitions, with flip angle=15°, TR=100ms, matrix=256x256, FOV=30cm, readout BW=±16kHz.

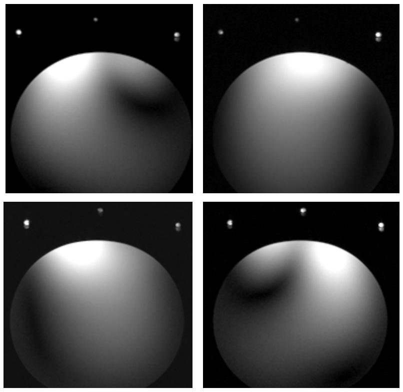

Figure 1: Gradient echo reception images from individual coils (left and right) from Scanner I (above) and II (below); white dots are vitamin capsule fiducials (cf Ref. 5), details in text above.

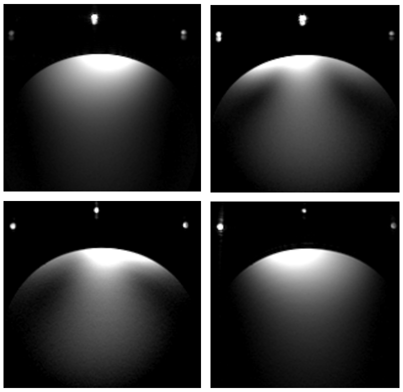

Figure 2: Gradient echo T/R images from the T/R coil pair – Above Scanner I, below Scanner II. Images at left (top & bottom) with transmitter at left; likewise images at right with transmit on right. Further details in Ref. 3.

$$\tilde{\chi} = -j\chi'' \begin{bmatrix} 1 & -j & 0 \\ j & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{[Eq. 1.]}$$

$$\tilde{Y} = R^{-1} \begin{bmatrix} 1 & j\eta \\ j\eta & 1 \end{bmatrix} \qquad \text{[Eq. 2.]}$$

References:
1. R. F. Harrington and A.T. Villeneuve, IRE Trans. Microwave Theory and Technique, **MTT6**, 308 (1958). 2. J. Tropp, Proc 15th ISMRM 2007, 320. 3. J. Tropp, Phys. Rev. A, **74** 062103 (2006). 4. P. B. Roemer, *et al.* Magn. Reson. Med. **16**, 192 (1990). 5. J. Tropp and T. Schirmer, J. Magn. Reson. **151**, 146 (2001).