3D MRI-based Electric Properties Tomographie Reconstruction using Volume Currents in the Method of Moments

C. Findeklee[1], and U. Katscher[1]

[1]Sector Medical Imaging Systems, Philips Research Europe, Hamburg, Germany

Introduction: The distribution of the electric conductivity σ and permittivity ε of human tissue can be used as diagnostic parameter and for a prediction of local SAR during MR measurements. Recently, a new approach called Electric Properties Tomography (EPT) was presented measuring these properties via an MR experiment [Kat06a,b]. The method is based on the $\underline{B}_1$-field distortions due to the electric properties of the tissue under test. For accurate reconstruction, 3D EM-field simulations of a detailed patient model have to be employed. This study investigates the use of Volume Currents in the Method of Moments (MoM) for EPT.

Theory EPT: From the z-component of Maxwell's equation $\mathrm{rot}\,\underline{\mathbf{H}}=\underline{\mathbf{J}}+j\omega\underline{\mathbf{D}}$, $\underline{\mathbf{D}}=\varepsilon\underline{\mathbf{E}}$ and $\underline{\mathbf{J}}=\sigma\underline{\mathbf{E}}$, we obtain

$$\sigma_c := \sigma + j\omega\varepsilon = (\partial_x \underline{\mathbf{H}}_y - \partial_y \underline{\mathbf{H}}_x)/\underline{\mathbf{E}}_z, \qquad (1)$$

assuming an isotropic σ_c. A positive z-directed B_0-field yields the transmit field rotating negatively around the z-axis: $\underline{B}_- = (\underline{B}_x - j\underline{B}_y)/2$. The other rotating component $\underline{B}_+ = (\underline{B}_x + j\underline{B}_y)/2$ is related to the receive sensitivity. Using standard $\underline{B}_1$-mapping techniques (e.g. [Sto96,Yar06]), the distribution of the absolute value $|\underline{B}_-|$ and the phase $\angle(\underline{B}_+ + \underline{B}_-)$ can be measured via MRI. Also $|\underline{B}_+|$ can be measured by negating $\underline{B}_0$. (This exchanges transmit and receive sensitivity and can be realized by rotating the object and coil inside the bore.) The Cartesian components are derived as $B_x = B_+ + B_-$ and $B_y = -jB_+ + jB_-$. Missing components of $\underline{B}_1$ have to be neglected, estimated (e.g. $\angle\underline{B}_+ \approx \angle\underline{B}_-$), or can be taken from 3D-field simulations, which are performed in each iteration anyway. With an initial estimation of the complex conductivity distribution $\sigma_c := \sigma + j\omega\varepsilon$, an estimation of the electric field component $\underline{\mathbf{E}}_z$ can be calculated. Subsequently using (1), a new estimation of $\sigma + j\omega\varepsilon$ and thus for $\underline{\mathbf{E}}_z$ is derived. This is iterated, until a stable distribution of the complex conductivity is obtained.

Theory MoM [Wan91]**:** The same Maxwell-equation as mentioned above can be written as

$$\mathrm{rot}\,\underline{\mathbf{H}} = \underline{\mathbf{J}}_{eq} + j\omega\varepsilon_0\underline{\mathbf{E}}, \qquad (2)$$

$$\underline{\mathbf{J}}_{eq} = (\sigma + j\omega(\varepsilon - \varepsilon_0))\underline{\mathbf{E}} \qquad (3)$$

$\underline{\mathbf{J}}_{eq}$ is the so-called (equivalent) volume current density, describing the current density of the bounded ($\sigma\underline{\mathbf{E}}$) and unbounded ($j\omega(\varepsilon-\varepsilon_0)\underline{\mathbf{E}}$) charges. Also Gauss' law can be written with the same $\underline{\mathbf{J}}_{eq}$:

$$j\omega\,\mathrm{div}(\varepsilon_0\underline{\mathbf{E}}) = -\mathrm{div}\underline{\mathbf{J}}_{eq} \qquad (4)$$

Equations (2,4) mean, that a conducting dielectric tissue can be modeled by free space, if the current distribution (3) is assumed instead. For numerical calculations, the volume is divided into small segments (cubes), in which a constant current density – described by three complex unknowns $\underline{J}_n$ – is assumed ($\underline{\mathbf{J}}_{eq} = \Sigma\, \underline{J}_n \cdot \mathbf{b}_n$, $\mathbf{b}_n$ are the so called base functions). From the free space Green's function $g(\mathbf{r},\mathbf{x}) = \exp(-jk|\mathbf{x}-\mathbf{r}|)/|\mathbf{x}-\mathbf{r}|$, $k=\omega c^{-1}$, the electric and magnetic field at position $\mathbf{x}$ can be calculated via integral equations from the equivalent currents at $\mathbf{r}$:

$$4\pi/\mu_0 \cdot (\underline{\mathbf{E}}(\mathbf{x}) - \underline{\mathbf{E}}_0) = \iiint \underline{\mathbf{J}}_{eq}(\mathbf{r}) \cdot g(\mathbf{r},\mathbf{x})\, dV + k^{-2}\iiint \mathrm{div}_r\, \underline{\mathbf{J}}_{eq}(\mathbf{r}) \cdot \nabla_{\mathbf{x}}\, g(\mathbf{r},\mathbf{x})\, dV \qquad (5)$$

$$4\pi \cdot (\underline{\mathbf{H}}(\mathbf{x}) - \underline{\mathbf{H}}_0) = \iiint \underline{\mathbf{J}}_{eq}(\mathbf{r}) \cdot \mathrm{rot}_{\mathbf{x}}\, g(\mathbf{r},\mathbf{x})\, dV \qquad (6)$$

$\underline{\mathbf{E}}_0$ and $\underline{\mathbf{H}}_0$ are the external excitation fields generated by an RF-coil. Matching (3) in the center of each voxel (so-called point matching) yields one linear equation per unknown current amplitude J_n, forming a set of linear equations for the unknown equivalent current amplitudes. This determines the calculation time per iteration step.

Methods/Results: The MoM-calculation was started with a homogeneous distribution of the complex conductivity $\sigma_{c,0}$, yielding an initial electric field $\underline{\mathbf{E}}_0$. Equation (1) was evaluated assuming the true transversal magnetic field distribution to be known from measurements (in this case simulated with the real σ_c). The described two steps were iterated, until a stable σ_c -distribution was achieved. The algorithm was tested using an $(9\text{mm})^3$ cube-Phantom, discretized with 9^3 equal-sized voxels, yielding 2187 unknown current amplitudes at the 3T-Larmor frequency of 127.728MHz. The 729 complex conductivities were reconstructed using a loop-coil of 10cm diameter with one resonance capacitor placed 20.5mm above the phantom surface (Fig.1). The current distribution on this coil and the excitation fields $\underline{\mathbf{E}}_0$ and $\underline{\mathbf{H}}_0$ were calculated with [CON] and assumed to be unaffected by the object. The conductivity σ was assumed growing linearly from 0.05S/m to 0.45S/m in x-direction, while the relative dielectric constant ε_r grows in y-direction from 10 to 90. At the 3^3 center elements, a tumor model was used with σ=0.5S/m and ε_r=100. Fig. 2 shows the starting values and the reconstructed σ and ε_r distributions in the nine slices for iterations 1,10,100 and 1000 with the corresponding mean relative error, mean($|\sigma_{c,n} - \sigma_c|/\sigma_c$).

Conclusion: The simulations show, that Volume Currents in the Method of Moments are a fast and stable approach for 3D EPT-reconstruction. A resolution of at least 25 voxels per skin depth $(\omega\sigma\mu_0)^{-0.5}$ should be used. A finite electric field is required to ensure the stability of the reconstruction. Since the equation systems are well conditioned (condition number around 56), fast iterative solvers can be used, and the 3D-reconstruction for EPT becomes feasible in reasonable time.

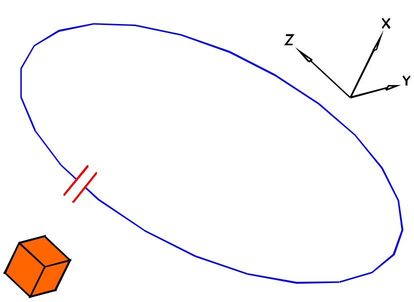

Fig.1: Geometric setup of simulated EPT Experiment

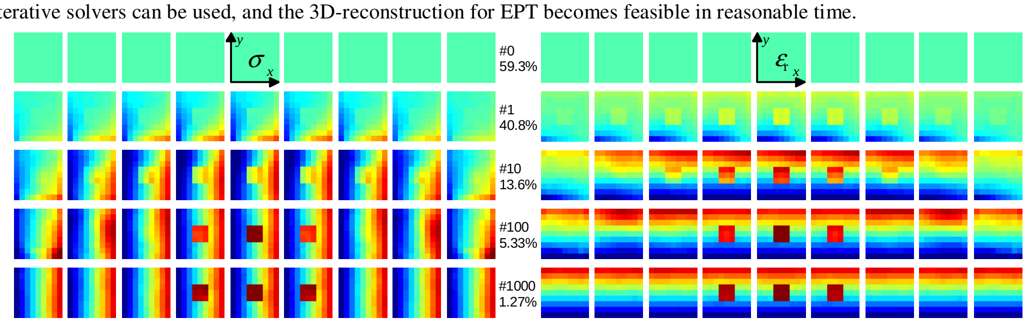

Fig.2: Starting values and reconstruction of conductivity (0-0.5S/m) and relative dielectric constant (1-100) for iterations 1, 10, 100 and 1000 - within each row, the nine slices of the cubic phantom are shown.

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