Modal Sound Radiation from Finite Cylindrical Shells

T. R. Lin[1], P. O'Shea[1], and C. K. Mechefske[2]

[1]Engineering Systems, Queensland University of Technology, Brisbane, Queensland, Australia, [2]Mechanical and Materials Engineering, Queen's University, Kingston, Ontario, Canada

Introduction

An unwanted side effect of increasing gradient coil current switching speed and the static magnetic field strength in MRI scanners is the increasing acoustic noise within the gradient coil and in the surrounding environment. The increased noise poses a health hazard to patients and healthcare workers. To control the noise radiated by the gradient coil, it is important to have a good understanding of the noise generation and control mechanisms of the structure. Modal sound radiation of cylindrical shells is typically approached by either assuming the cylindrical shells are infinite in extent [1, 2], or by employing statistical methods [3]. However, the radiated sound field at low and medium frequency ranges is usually dominated by the sound pressure radiated by only a few modes and the statistical approach is not applied. Furthermore, sound radiation of a short cylindrical shell (a gradient coil) is affected by the boundary conditions and by the interference between the internal and external sound fields. The relationship between modal radiation of a short cylindrical shell and that of a corresponding infinite shell is largely unknown, and so is investigated in this paper. This investigation is done in the hope that the findings can improve our understanding of sound radiation from finite cylindrical shells and subsequently lead to better noise control in MRI gradient coils and other pertinent industry applications.

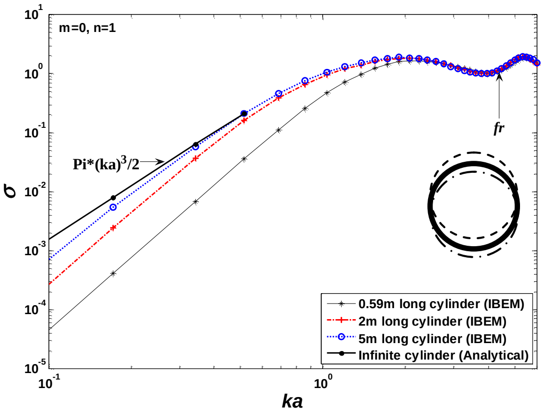

Figure 1. Modal radiation efficiency of (0, 1) cylindrical mode

Method

The gradient coil cylinder model used by Wang and Mechefske [4] is adopted for this study. The cylinder is 0.59m long and has a mean radius of 0.188m, with uniform 11.5mm shell thickness. By definition, the cylinder model is an acoustical thin cylindrical shell whose critical frequency is far greater than the ring frequency ($f_c > f_r$). Free-free boundary conditions are assumed for the cylinder in the simulation. The modal information of the cylinder is extracted from the normal mode analysis provided by finite element analysis using MSC/NASTRAN. The modal radiations of two effective sound radiation modes (Mode (0, 1), a dipole source and Mode (0, 2), a quadruple source) of the finite cylinder are calculated by the indirect boundary element method using LMS-Virtual.Lab. The procedure is repeated for another two cylinders of different lengths (one is 2m long and another 5m long). The modal radiation efficiency is calculated by:

$$\sigma = \frac{W_{rad}}{W_{in}} , \qquad (1)$$

where $W_{rad} = \frac{1}{2}\int_S \mathrm{Re}(p v_n^*)\, dS$ and $W_{in} = \frac{1}{2}\rho_0 c \int_S \left| v_n^2 \right| dS$ are respectively the radiated sound power and the input power associated with the modal vibration, p is the acoustic pressure, v_n is the normal surface velocity of the gradient coil cylinder and $\rho_0 c$ is the characteristic impedance of air.

Modal radiation efficiencies of infinite cylindrical shells at low frequencies are given by [1]:

$$\sigma_n \approx \frac{4\pi}{(n!)^2}\left(\frac{ka}{2}\right)^{2n+1}, \qquad n \geq 1,\ ka < \frac{n}{2} \qquad (2)$$

where $k = \omega / c$ is the acoustic wave number, ω is the angular frequency, c is the wave speed in air, a is the mean radius of the cylindrical shell, and n is the modal index of the cylinder in the circumferential direction.

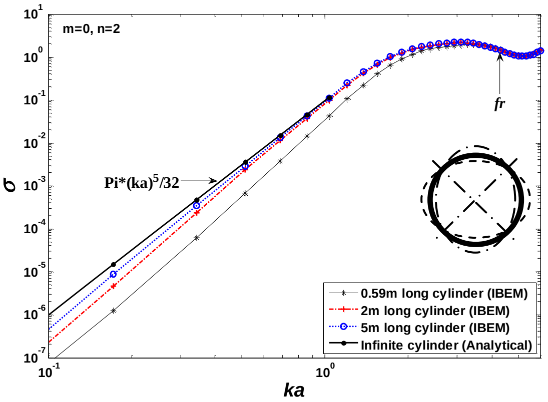

Figure 2. Modal radiation efficiency of (0, 2) cylindrical mode

Results and Discussion

Modal radiation efficiencies of the two circumferential modes of the finite cylinders and the corresponding infinite cylinder are shown in Figures 1 and 2, respectively. In contrast to that of flat plate panels, modal radiation efficiencies of the finite cylindrical shells can be divided into three regions, $f < f_r$, $f_r < f < f_c$ and $f > f_c$. At low frequencies, modal radiation efficiencies of finite cylindrical shells approach those of the corresponding infinite shell as the length of the cylinder or the ratio between the cylinder length and the acoustic wavelength increase. In this frequency range, the modal radiation efficiencies of the corresponding infinite cylindrical shell serve as the upper bounds to those of finite cylinders. The radiation efficiencies of the two cylindrical modes increase rapidly (a 30dB/decade and 50dB/decade increase, respectively) and become greater than unity at frequencies well below the ring frequency. After the first peak, the radiation efficiency decreases as the frequency increases further and approaches unity at frequencies around the ring frequency. Above the ring frequency, the curvature effects on flexural wave speed disappear [2] and the sound radiation of the cylindrical shell is similar to that of flat plate panels. As a result, the modal radiation efficiency increases again and reaches another peak as the frequency approaches the critical frequency. Similar to that of flat plate panels, the radiation efficiency approaches unity after the critical frequency.

Conclusion

The modal radiation efficiency of finite cylindrical shells is shown to approach asymptotically that of the corresponding infinite cylindrical shell as the ratio of the cylinder length to acoustic wavelength increases. It was found that the modal radiation efficiency of an acoustical thin finite cylindrical shell can be divided into three regions by the ring and critical frequencies and has two radiation peaks.

References

1. Cremer, L, et al, Structure-Borne Sound, Springer 2004. 2. Fahy, F, Sound and Structural Vibration, Academic Press 1986. 3. Szechenyi, E, J. Sound and Vib. 1971 Vol. 19: 65-81. 4. Wang, F, CK Mechefske, Concepts in Magn. Resonance Part B 2005, Vol. 27B: 34-50