Transmit B1 Shimming at High Field with SAR Constraints: A Two Stage Optimization Method Independent of The Initial Set of RF Phases and Amplitudes

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Introduction: There has been an increasing interest in constraining transmit B₁ shimming with specific absorption rate (SAR) limits [1-2], especially at high magnetic field. Since most of the existing methods rely on solving a nonconvex optimization problem, they are typically faced with two difficulties: Only local optimum solutions are obtained, and they are susceptible to the chosen initial points for optimization. Here we introduce a two stage optimization method where a reliable initial point is acquired in the first stage by a convex semidefinite relaxation (SDR) approximation method. A high quality B₁ shimmed map then can be obtained in the second stage optimization using the SDR initial points. The presented technique is verified with simulations for a 16-channel transmit coil array at 7T with a human head model.

Proposed Method: Denote by $\mathbf{x} \in \mathbb{C}^{16}$ the complex vector where each entry defines the magnitude and phase of each RF coil. Let $\mathbf{a}_i = [a_{i,1}, \ldots, a_{i,16}]^T \in \mathbb{C}^{16}$ where $a_{i,j} \in \mathbb{C}$ represents the B₁ field magnitude and phase at sample i due to the jth RF coil. Then the B₁ field magnitude at the ith sample is given by $|\mathbf{a}_i^T \mathbf{x}|$. By defining a B₁ target map with uniform magnitudes at each sample (i.e., pixel) equal to $b > 0$, we consider the following optimization problem for homogeneous B₁ shimming with an average SAR constraint

$$\min_{\mathbf{x} \in \mathbb{C}^{16}} \left\{ \max_{i=1,\ldots,m} \left| |\mathbf{a}_i^T \mathbf{x}|^2 - b^2 \right| \right\} \quad \text{subject to (s.t.) } \mathbf{x}^H \mathbf{G} \mathbf{x} \leq \rho, \qquad (1)$$

where m denotes the number of samples in the B₁ map, $\rho > 0$ is a preset number, and $\mathbf{x}^H \mathbf{G} \mathbf{x}$ denotes the average SAR in which $\mathbf{G} \in \mathbb{C}^{16\times 16}$ is composed of the complex valued E field coefficients, and of the tissue conductivity and mass density [3]. It can be seen from (1) that the proposed criterion tries to make the combined map as uniform and as close to the target map as possible. Since problem (1) is a nonconvex problem, usually a local minimum solution is obtained and it is highly dependent on the chosen initial points. Here we present a two stage optimization method for (1) in which a reliable initial point is first obtained in the first stage through a SDR approximation method (see Fig. 1). To illustrate this, let us define $\mathbf{X} = \mathbf{x}\mathbf{x}^H$ [which is equivalent to $\mathbf{X} \succeq 0$ (Hermitian and positive semidefinite) and $\mathrm{rank}(\mathbf{X}) = 1$]. By writing problem (1) in terms of $\mathbf{X}$ and dropping the nonconvex rank-1 constraint, one can obtain the following SDR of problem (1)

$$\min_{\mathbf{X} \in \mathbb{C}^{16\times 16}} \left\{ \max_{i=1,\ldots,m} \left| \mathrm{trace}(\mathbf{a}_i^* \mathbf{a}_i^T \mathbf{X}) - b^2 \right| \right\} \quad \text{s.t. } \mathrm{trace}(\mathbf{G}\mathbf{X}) \leq \rho, \; \mathbf{X} \succeq \mathbf{0}. \qquad (2)$$

Different from problem (1), the relaxation problem (2) can be shown to be a convex optimization problem and can always be efficiently solved with global minima. The comparison of problems (1) and (2) is summarized in Table I. Let $\mathbf{X}^\star \in \mathbb{C}^{16\times 16}$ be the optimum solution of problem (2). An approximate solution of problem (1) based on $\mathbf{X}^\star$ can be obtained by the following randomization procedure: We generate L random vectors $\xi^{(\ell)} \in \mathbb{C}^{16}$, $\ell = 1, \ldots, L$, from the complex Gaussian distribution $\mathcal{N}_c(\mathbf{0}, \mathbf{X}^\star)$. Let

$$\mathbf{x}^{(\ell)} = \begin{cases} \xi^{(\ell)}, & \text{if } (\xi^{(\ell)})^H \mathbf{G} \xi^{(\ell)} \leq \rho, \\ \xi^{(\ell)} / \sqrt{(\xi^{(\ell)})^H \mathbf{G} \xi^{(\ell)}} \cdot \sqrt{\rho}, & \text{otherwise}, \end{cases}$$

$$\ell^\star = \arg \min_{\ell=1,\ldots,L} \left\{ \max_{i=1,\ldots,m} \left| |\mathbf{a}_i^T \mathbf{x}^{(\ell)}|^2 - b^2 \right| \right\}.$$

An approximate solution of problem (1) can be obtained by $\mathbf{x}_{\mathrm{sdr}} = \mathbf{x}^{(\ell^\star)}$. This approximate solution however can be taken as an initial point of problem (1) for further optimization, and thus we propose in the second stage to solve problem (1) using nonlinear programming techniques, as illustrated in Fig. 1. It might be argued that an initial point of problem (1) can be obtained by randomly generating a set of feasible vectors from the distribution $\mathcal{N}_c(\mathbf{0}, \mathbf{I}_{16})$, and choosing the one with minimum objective value. This ad-hoc method is similar to the above randomization procedure, but the covariance matrix of the complex Gaussian distribution is replaced by the 16 by 16 identify matrix. It will be demonstrated in our simulations that the initial points obtained from the SDR method are actually more reliable compared to the ad-hoc random initializations.

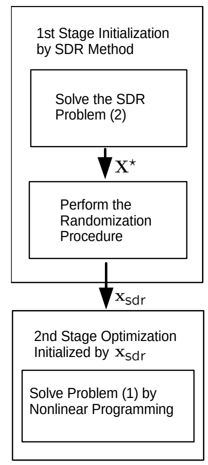

Fig. 1 Block diagram of the proposed two stage optimization method

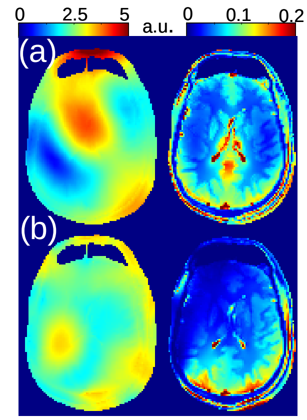

Fig. 2 B₁ magnitude maps (Left) and SAR maps (Right) of (a) non-optimized weights and of (b) optimized weights for $\rho = 0.05$.

Table I Comparison of problems (1) and (2)

Problem (1)	nonconvex and has many local minima	The optimized results depend on the chosen initial points.
Problem (2)	1. convex and only has global minima 2. efficiently solvable by interior point methods [4].	The initial points do not affect the optimized results.

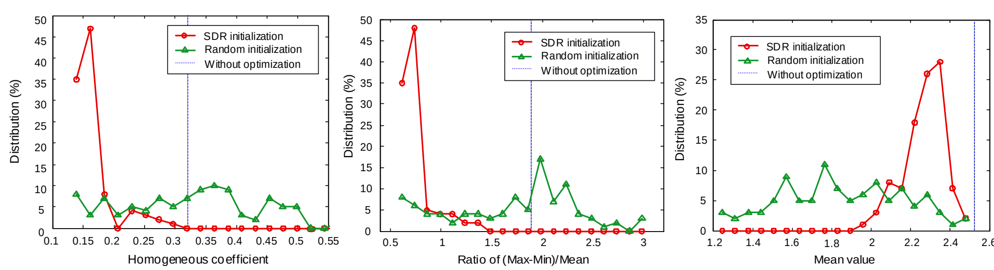

Fig. 3 Distributions of (a) homogeneous coefficient (b) flatness coefficient and (c) mean value of optimized B₁ maps using SDR initializations and using randomly generated initial points.

Simulation Results and Discussions: In our simulations, the coil used in the model is a 16-element RF strip line coil array [5] mounted on a cylindrical former of 32cm in diameter and loaded with a human head. The B₁ and E field maps in the brain were simulated with the XFDTD software (REMCOM Inc.). The SeDuMi [6] was employed to solve problem (2), while problem (1) in the second stage was solved by the optimization routine provided in MATLAB (MathWorks Inc.). The number of randomization vectors was set $L = 500$. Figure 2 shows the results for (a) non-optimized weights, i.e., $\mathbf{x} = [1, e^{j2\pi/16}, \ldots, e^{j30\pi/16}]^T$ (corresponding to the geometric azimuthal phase distribution for 16 channels) and for (b) optimized weights for average SAR constraint $\rho = 0.05$ and the magnitude of target map b equal to the mean value of B₁ magnitudes in Fig. 2(a). The optimized weights in Fig. 2(b) were scaled such that the mean value of associated B₁ magnitudes is equal to that in Fig. 2(a). It can be seen that a more uniform B₁ magnitude map is obtained while the SAR is significantly reduced (compared to non-optimized result in Fig2 (a)). To demonstrate the robustness of the SDR initializations, we compared it with the ad-hoc random initialization method by performing 100 trials of simulation using different seed settings in each trial for the random vector generation. Figure 3 shows the corresponding distributions of (a) the homogeneous coefficient (defined as the ratio of standard deviation and the mean value of B₁ magnitudes), (b) the flatness coefficient (defined as the ratio of the difference between the maximum and the minimum B₁ magnitudes and the mean value of B₁ magnitudes) and (c) the mean value of B₁ magnitudes under average SAR constraint $\rho = 0.05$. One can see that for the SDR initialization there is at least 95% probability to obtain a shimmed B₁ map with flatness coefficient less than 1.25. In conclusion, the presented optimization criterion in (1) together with the proposed two stage optimization method provides a new B₁ shimming technique which features its insensitivity to the initial points while high quality B₁ shimming can be achieved. **References:** [1] A. T. Cornelis, *et al.*, MRM 2007, 57:577-586. [2] Z. Wang, *et al.*, ISMRM 2007, p.1022. [3] Y. Zhu, MRM 2004, 51:775-784. [4] S. Boyd *et al.*, *Convex Optimization*, Cambridge, Univ. Press 2004. [5] G. Adriany, *et al.*, MRM 2005, 53(2):434-445. [6] J. F. Sturm, Opt. Methods. and Software 1999, 11-12:625-653. **Acknowledgements:** U.S. NSF Grant DMS-0610037, R01-MH070800, and BTRR-P41-RR008079, P30-NS057091.