Q measurement and simulation for RF coils

G. Jacobs[1], A. Assefa[1], and J. Willig-Onwuachi[1]

[1]Department of Physics, Grinnell College, Grinnell, Iowa, United States

Introduction: A widely used figure of merit in radiofrequency (RF) coil construction and testing is the Q or 'quality factor.' Q is a measure of the spectral selectivity of the coil, $Q = f_0/\Delta f$, where f_0 and Δf are the central frequency and bandwidth (FWHM) of the resonance, respectively. A narrow bandwidth (high Q) excludes more noise and results in better signal to noise ratio. Q is also related to the ratio of stored energy to the energy dissipated per cycle, giving an alternate definition of $Q = \omega_0 L/R$. While the theory behind Q and what it means is well established, determining the best way to measure it for RF coils is not as clear. It is not always straightforward in practice. Considering how often coil Q is reported in the literature (i.e. for a new coil), there is surprisingly little literature on how to accurately measure Q. There are a variety of techniques to choose from (see, e.g. [1-3]). Also, simply connecting the coil to a measurement device such as a network analyzer (NA) (an instrument for measuring transmitted or reflected power as a function of driving frequency) perturbs the measurement by adding a load. In this case, a coupling coefficient, which compares the power dissipated externally to that dissipated internally, relates the loaded and unloaded Q (Q_L and Q_0 respectively): $Q_0 = Q_L(1+\kappa)$. Note that the use of the terms 'loaded' and 'unloaded' Q in the theoretical literature is related to but distinct from the common use of these terms experimentally (where the load is a result of coupling to the anatomy or sample being imaged).

This project is an initial attempt to compare several common methods of measuring Q with each other and with expected theoretical results. Matlab was used to model resonant circuit behavior by calculating complex impedance and the reflection coefficient. Several small square loop coils were constructed and characterized. Then the Q of each was measured using the various methods and the theoretical Q was computed.

Methods: Five square coils were built (ranging from 11-15 cm across) using etched circuit boards or copper tape and were connected to the NA either inductively or directly. For direct coupling, a BNC cable was connected across one of the coil capacitors with a single matching capacitor (in series with the inner wire). For inductive coupling, a search loop (semi-rigid coax cable with inner wire looped and connected back to its own shield, which has a gap in it to allow a voltage difference to be measured) was connected to the NA and held above the coil. Magnetic flux caused by the alternating current in the search loop induces a current in the coil, resulting in a reduction in reflected power (single loop reflection measurement) or an increase in flux measured by a second loop (two loop transmission measurement).

Several variations of both transmission and reflection based measurement techniques were used. A 2-port transmission test was used in each of three configurations: 1) one direct connection and one search loop, 2) two search loops decoupled by a large distance, and 3) two search loops decoupled by overlapping. In each case, the center frequency and FWHM (-3dB bandwidth) of the resonance peak were measured (see Fig. 1). Also, a 1-port reflection test was used to map out the reflection coefficient in the complex plane (this yields a circle that advances clockwise with frequency, see Fig. 2). The center frequency, f_0, and frequencies at ±45° from f_0 (other points may be used, 45° simplifies the math) were determined and used to calculate the coupling coefficient and unloaded Q from the size of the circle [1].

Six different measurements were performed for five separate coils. **R1** and **R2** are single loop inductive measurements with Q_0s averaged over 3 distances (search loop locations); the difference is that R1 is *just* the coil and R2 has a matching capacitor and a 50 ohm terminated cable. **R3** is from a single direct connection. A data file containing the reflection coefficients and associated frequencies was run through an algorithm that fits it to a circle (Richard Brown, 'fitcircle', 2007) which is then used to find Q. **T1** and **T2** use two search loops, the former decoupled by distance and the latter by overlapping. **T3** uses one direct connection and one search loop. These three transmission tests yield Q_L and require κ to determine Q_0.

Simulations were performed using MATLAB 7.1 on a Pentium IV workstation. Capacitance values for simulation were taken from quoted manufacturer specifications. The resonance frequency, f_0, was measured with a search loop and the NA and used along with the capacitor values to compute the coil inductance. Resistance values were found by trial and error or estimated by a separate measurement of the coil resonance with nothing attached ($R=\omega_0 L/Q_0$) using a reflection test. For cases with a cable attached to the coil, it is important to simulate the effects of the cable as well. A simple transmission line model was used to account for cable length and a series capacitor was used to approximate the cable capacitance.

Results and Discussion: Our simulation model replicated experimental coil behavior well. An example simulation shown in Fig. 3 matches the corresponding data in Fig. 2 closely. Note that for direct connect measurements, it is important to incorporate a model for the transmission line.

Table 1 shows strong agreement between measurements R1, R3, T1, T2, and Theory. It also indicates that measurements R2 and T3 agree well. While it seems odd for T1 and T2 (which are Q_Ls) to agree with R1 and R3 (Q_0s), the coupling coefficient for a search loop held far away is negligible and consequently $Q_L \approx Q_0$. T3 has two different κ's; the search loop's κ may be ignored, but the direct connect still loads the coil and must be accounted for to reach the same Q_0. Finally, though R2 seems to give a different number, this 'Q_0' is actually measured with a 50 Ω load (the Q_L for a direct connection). Thus, all of these measurements agree when everything is taken into consideration.

Some additional measurements taken with the coils coupled to either a human leg or a bottle of water helped reveal what Q_0 is and is not. It is *not* the Q measured whenever the coil is in free space (not coupling to a leg or a water bottle). It is *not* a value independent of coil location. The numbers differ even when rematched, retuned, and the coupling coefficient is taken into account. Q_0 is simply the Q one would measure if the measuring device were not adding a load to the coil.

For inductive measurements, a search loop that was slightly smaller than the coil seemed to work best (a small loop will not pick up much signal and a large loop results in resonance shifting). Also, regular recalibration of the network analyzer and adjustment of the frequency span (ideally, a small span centered at f_0) is necessary for good data acquisition. Optimal results using the 2-loop transmission method were obtained while coupling weakly (just above the noise level).

There are many available methods of Q-factor measurement. Some, notably those intended only for very high Q systems, such as a microwave cavity resonator, were not examined in this study and may require further study. This study focused on common transmission and reflection methods. Even though there are numerous ways to carry out these two tests, our findings suggest that, when everything is accounted for, all methods examined, when properly used, should lead to the same value and should approximately agree with theory. Since direct connections are generally lossier than inductive connections and transmission tests require only the network analyzer (no outside computer and code), it is our opinion that **T2** (search loops decoupled through overlapping) is the most efficient measurement. Please note that this measurement will only be close to Q_0 if the loops are very well decoupled (-70 or -80 dB when isolated from the coil) and if the signal is just above the noise.

References: [1] Darko Kajfez, *Q Factor*, 1994, [2] Youn-Ho Chuong, PhD Thesis NCSU, 1979, [3] Rachan Soonthornmuang, MS Thesis U of Mississippi, 1998.

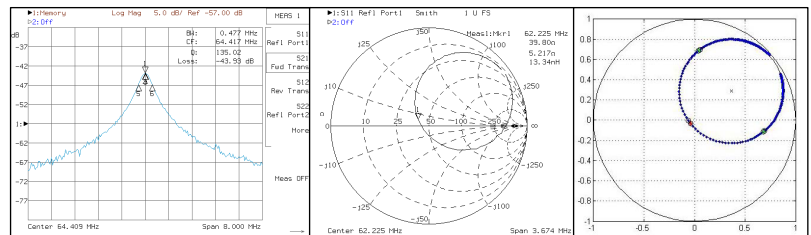| Table 1	Q_o	Q_o	Q_o	Q_o	Q_L	Q_L	Q_L
	Theory	R1	R2	R3	T1	T2	T3
11cm	220	220	110	200	215	217.5	97.5
13cm	255	265.5	142	254.5	267.5	272.5	132.5
15cm	227	230	117	240	235	233	117.5
Etch	245	242.5	125	230	232.5	235	117.5
Thin	238	248	136	249	245	242.5	124.5

Figure 1   Figure 2   Figure 3