Simple Analytical Equation of the Induced E-Field

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INTRODUCTION:

During MRI, the time-varying gradient magnetic field may cause stimulation at the nerves or muscles by inducing an electric field. In order to define this induced electric field, both computational and analytical calculations have been performed in the literature using inhomogeneous and homogenous human models [1, 2]. The effect of the electric and magnetic fields inside the body with an active implantable medical device during MRI was also investigated [3]. In the same study, to observe the worst magnetic field direction corresponding to the implant position, and to define the worst distance between the nerve and implant tip causing the nerve stimulation, uniform magnetic field was taken over the cross section of torso, and the maximum induced E-field has been calculated, accordingly. In our study, the E-field in a cylindrical object is calculated assuming that a spatially dependent magnetic field is produced by uniform gradient fields, and to reach a more general and understandable expression of the E-field for this magnetic field, the obtained result is simplified. With these result, the worst position causing the stimulation for small implants can easily be defined.

THEORY:

As a general convention, magnetic fields used for imaging are defined in the z-direction*;* however gradient fields occur in all three directions and all these alternating magnetic field components contribute to the electric field induction in the body. Assuming that induced currents in the body do not alter the magnetic field significantly, the curl of the magnetic field is taken as zero ($\nabla\times\vec{B}=0$) within the cylindrical body [4]. Using this expression and given B_z=G_x(t)x +G_y(t) y +G_z(t)z, the remaining components of magnetic field, i.e. B_x and B_y can be calculated, and the total magnetic field is given by [4]:

$$\vec{B}(x,y,z,t)=G_x(t)\left\{z\hat{a}_x+x\hat{a}_z\right\}+G_y(t)\left\{z\hat{a}_y+y\hat{a}_z\right\}+G_z(t)\left\{-0.5x\hat{a}_x-0.5y\hat{a}_y+z\hat{a}_z\right\}, \quad (1)$$

where, $G_x(t)$, $G_y(t)$ and $G_z(t)$ are the gradient fields and, $\hat{a}_x$, $\hat{a}_y$, and $\hat{a}_z$ are the unit vectors in x, y and z directions, respectively.

In order to obtain the electric field equation, cylindrical homogenous volume with radius ρ_0 and conductivity σ, is assumed. The same solutions were found as Bowley and Bowtell [2] by using the quasi-static assumption. They formed the electric field equation with respect to the current passing through the coils and observed different electric and magnetic field pattern graphs by changing the radii of the cylindrical coil and the conductive volume. However, to obtain these graphs, integrals and summations are numerically evaluated, which shows that these resultant expressions are incomprehensible. In our study, these analytical equations are simplified by evaluating these integrals and summations using low frequency based approximations. First, using the B_z field given in equation (1), the current passing through the coil is defined. Using this current, the non-zero terms of the summation are determined. For the frequency range where the gradient waveforms are used, wavelength is significantly smaller than the body size. As a result, Bessel functions in these expressions are turned into some simple algebraic functions and hence related integrations and summations are performed in closed form.

RESULT:

The resultant, simplified E field equation for x, y, and z coils are given as following:

$$\vec{E}(x,y,z,t)=G_x'(t)\left\{0.5xy\hat{a}_x+0.25\left(-\rho_0^2+y^2-x^2\right)\hat{a}_y-yz\hat{a}_z\right\}+G_y'(t)\left\{0.25\left(\rho_0^2+y^2-x^2\right)\hat{a}_x-0.5xy\hat{a}_y+xz\hat{a}_z\right\}+G_z'(t)\left\{0.5yz\hat{a}_x-0.5xz\hat{a}_y\right\}, \quad (2)$$

where ρ_0 is the radius of the cylindrical volume $G_x'(t)$, $G_y'(t)$ and $G_z'(t)$ are the first order time derivatives of the x, y and z coil gradient fields, respectively.

Note that this equation satisfies the divergence of E-field being equal to zero condition ($\nabla\cdot$E=0). This result is in line with our intuitive understanding on what electric field should be. At the surface of the object ($\rho=\rho_0$) the EM fields in cylindrical coordinates are given as:

$$\vec{E}(\rho=\rho_0,\phi,z,t)=G_x'(t)\left\{-0.5\rho_0^2\cos\phi\hat{a}_\phi-\rho_0 z\sin\phi\hat{a}_z\right\}+G_y'(t)\left\{-0.5\rho_0^2\sin\phi\hat{a}_\phi+\rho_0 z\cos\phi\hat{a}_z\right\}+G_z'(t)\left\{-0.5\rho_0 z\hat{a}_\phi\right\},$$

$$\vec{B}(\rho=\rho_0,\phi,z,t)=G_x(t)\left\{z\cos\phi\hat{a}_\rho-z\sin\phi\hat{a}_\phi+\rho_0\cos\phi\hat{a}_z\right\}+G_y(t)\left\{z\sin\phi\hat{a}_\rho+z\cos\phi\hat{a}_\phi+\rho_0\sin\phi\hat{a}_z\right\}+G_z(t)\left\{-0.5\rho_0\hat{a}_\rho+z\hat{a}_z\right\}.$$

In [5], to define the worst case for the electric field causing the stimulation, the z gradient magnetic field, which is normal to the circular cross section of radius ρ_0, was taken as uniform. In this study, the worst electric field result defined from simplified equation is compared with $|\vec{B}_z'(t)|=2|\vec{E}_\phi(r)|/\rho_0$, given in [5], where $\vec{B}_z'(t)$ is the first order time derivative of the B_z field. As a result, same expressions are observed. But in [5], only ϕ directed component of the E-field was given as the maximum E-field. However, maximum electric field has also component in the z direction, which is an important information to define the path of the implant lead. Moreover, the maximum magnetic field and the maximum electric field locations are different as also reported in [6] by volunteer experiments.

CONCLUSION AND DISCUSSION:

In order to define simplified expression of electric field in a cylindrical object, uniform cylindrical gradient coils are assumed. With these simplified expressions, the electric field distribution inside the body can easily be understood without an additional computational work. In the case of an implant in the body, the induced current on the lead of the implants can be calculated using these formulation and moreover, the implant lead path with no stimulation can be determined.

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