# Magnetic Resonance Elastography Based Method for Quantitating Shear Stiffness within a Heart Simulating Phantom Using a Thin Spherical Shell Model

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## Introduction:

It has been suggested that cardiac dysfunction is related to the mechanical properties of the myocardium and that knowledge of this parameter could provide insight into a variety of diseases such as diastolic dysfunction [1], hypertension [2] and myocardial ischemia [3]. However, the ability to quantify myocardial tissue mechanical properties in vivo is currently limited. Our group has developed a noninvasive phase-contrast based MR imaging technique known as MR elastography (MRE) [4] that is capable of spatially resolving the shear modulus μ of tissue-like materials. Existing methods for calculating μ generally assumes that the shear wave is propagating in a uniform, infinite medium [5]. However, this assumption is not valid in the heart because of the heart's complex geometry - a fluid-filled chamber with comparatively thin walls. The purpose of this work is to propose a new MRE based method for estimating μ that includes boundary condition effects similar to those encountered within the heart based on shear wave propagation within a thin spherical shell.

**Theory:**

*Shear Wave Propagation in a Thin Spherical Shell:* The vibrations of a spherical shell include both membrane and flexural effects that result in wave propagation guided by the boundary conditions of the object. The equations of motion can be obtained by applying Hamilton's variational principle [6] and by assuming midsurface deflections and nontorsional axisymmetric motions. Expressed in the polar coordinate system these equations are described by Eqs. 1 and 2 where a = radius, u = circumferential component of displacement, w = radial component of displacement, $c_p$= flexural wave speed, $\beta = h^2/12a^2$, h=thickness of the shell, $p_a$=applied load, and E = Young's modulus. If the circumferential and radial displacements are known, Eqs. 1 and 2 can be solved for $c_p$. The shear modulus of the material can then be calculated according to the relationship: $\mu = 0.5*(1-\nu)\rho c_p^2$, where ρ = density and ν = Poisson's ratio.

$$(1+\beta^2)\left[\frac{\partial^2 u}{\partial\theta^2}+\cot\theta\frac{\partial u}{\partial\theta}-(v+\cot^2\theta)u\right]-\beta^2\frac{\partial^3 w}{\partial\theta^3}-\beta^2\cot\theta\frac{\partial^2 w}{\partial\theta^2}+[(1+v)+\beta^2(v+\cot^2\theta)]\frac{\partial w}{\partial\theta}-\frac{a^2\ddot{u}}{c_p^2}=0 \quad \textbf{(1)}$$

$$\beta^2\frac{\partial^3 u}{\partial\theta^3}+2\beta^2\cot\theta\frac{\partial^2 u}{\partial\theta^2}-[(1+v)(1+\beta^2)+\beta^2\cot^2\theta]\frac{\partial u}{\partial\theta}+\cot\theta(2-v+\cot^2\theta)\beta^2-(1+v)]u-\beta^2\frac{\partial^4 w}{\partial\theta^4}$$
$$-2\beta^2\cot\theta\frac{\partial^3 w}{\partial\theta^3}+\beta^2(1+v+\cot^2\theta)\frac{\partial^2 w}{\partial\theta^2}-\beta^2\cot\theta(2-v+\cot^2\theta)\frac{\partial w}{\partial\theta}-2(1+v)w-\frac{a^2\ddot{w}}{c_p^2}=-p_a\frac{(1-v^2)a^2}{Eh} \quad \textbf{(2)}$$

In this application MRE techniques were used to quantify and spatially resolve the circumferential and radial components of a propagating wave in thin spherical shell phantoms, and then solve for μ using the aforementioned techniques.

**Methods:**

*Phantom Experiments:* Two spherical phantoms were constructed using silicone rubber (Wirosil, BEGO, Germany) poured as spherical shells of varying thicknesses and inner diameters (98mm, 100 mm diameter with thicknesses of 13.5 mm and 17 mm, respectively). MRE was performed on each shell, connected to a static pressure source and imaged on a 1.5T MR scanner (GE Health Care, Waukesha, WI). External motion was applied using an electromechanical tapper applied to the top of the phantom and driven by a sinusoidal waveform at 200 Hz. A gradient echo MRE acquisition was performed with TR/TE =150/16 ms, FOV = 140 mm, 30° flip angle, 10-mm slice thickness, and a 256x64 acquisition matrix. The horizontal and vertical components of the induced wave motion were measured during the MRE acquisition. These were then transformed into their radial and circumferential components for use in Eq. 2 using a Cartesian-to-polar coordinate conversion that assumes the center of the coordinate system to be at the centroid of the acquired phantom images. The displacement fields were then processed for $c_p$ using a direct inversion of Eq. 2 and assuming the load $p_a$ to be negligible. Derivative estimates were provided by Savitsky-Golay filters [7] fit to data in adaptive windows conforming to the boundary of the object and approximately half of the flexural shear wave length. After spatially resolving $c_p$, μ was calculated as above.

*Computer Simulation:* To determine whether or not the aforementioned MRE-based approach can accurately map the circumferential and radial components of the propagating shear wave as described in Eqs. 1 and 2, an average value of μ was calculated from the MRE-derived shear stiffness map for the 98 mm inner diameter phantom. $c_p$ was calculated from this value which was then input into previously derived solutions to Eqs. 1 and 2 under the conditions of forced axisymmetric nontorsional vibrations [6], providing radial and circumferential displacement fields. The MRE-derived and simulated displacement fields were then compared to determine the degree of agreement between the measured and theoretically predicted displacement values.

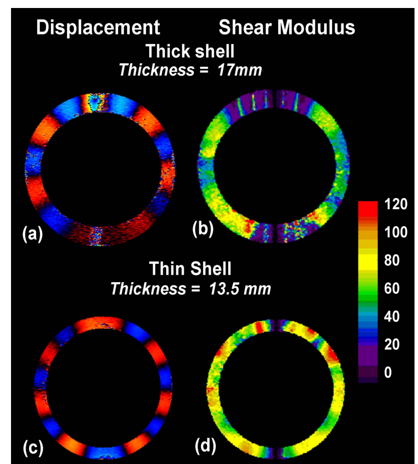


**Figure 1: Radial displacement field in a thick (a) and thin shell (c); Shear stiffness (kPa) in thick (b) and thin shell (d).**

**Results:**

Figure 1(a) shows the radial component of the flexural wave displacement field for the 17 mm thick shell. Figure 1(b) shows the shear modulus calculated by solving Eq. 2 for this shell. Figure 1(c) shows the radial component of the flexural wave displacement field for the 13.5 mm shell and figure 1(d) shows the calculated shear stiffness map. The mean shear stiffness of these phantoms was calculated to be 65±17 kPa and 71±14 kPa for the 17 mm and 13.5 mm diameter shells respectively. Figure 2 compares the MRE-based measured and simulated displacement fields. The MRE-derived value of μ equal to 71 kPa was used to calculate a $c_p$ value of 16.85 m/s assuming $\rho$ = 1000 kg/m$^3$ and ν = 0.5. Figure 2(a) shows the radial component of the MRE measured displacement field and (b) shows the same displacement field derived from the solutions to Eqs. 1 & 2 [6] using the aforementioned parameters.

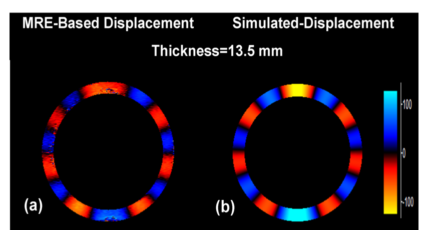


**Figure 2: (a) Radial component of displacement in a phantom (μm). (b) Simulated shell.**

**Discussion:**

These preliminary results indicate that MRE-based techniques can be used to spatially resolve circumferential and radial displacements induced by propagating shear waves in a thin spherical shell, thereby facilitating a more accurate MRE-based method for assessment of shear modulus in organs whose geometry can be modeled as a shell-like object. The organs for which this model may be applied include the heart, the eye, and the bladder.

**References:**

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