## Computational Fluid Dynamics Analysis of Bolus Dispersion in Myocardial Perfusion Measurements

## D. Graafen<sup>1</sup>, K. Münnemann<sup>1</sup>, S. Weber<sup>1</sup>, and W. G. Schreiber<sup>1</sup>

<sup>1</sup>Department of Diagnostic and Interventional Radiology, Section of Medical Physics, Mainz University Medical School, Mainz, Germany

### **Introduction**

Quantification of myocardial blood flow by means of dynamic  $T_1$ -weighed MRI requires the knowledge of the arterial input function (AIF). In first pass myocardial perfusion imaging the AIF is usually estimated from the left ventricle (LV) [1,2]. Dispersion of the contrast agent bolus may occur between the LV and the tissue of interest, e.g. as a result of the vascular flow profile, and/or at a stenosis. This effect can be described mathematically by a convolution of the LV AIF with a vascular transport function (VTF): AIF<sub>out</sub>(t) = AIF<sub>in</sub>(t)  $\otimes$  VTF(t) [3,4]. A quantitative parameter of the VTF which reflects the dispersion is the variance  $\sigma^2_{VTF}$  which can be calculated from AIF<sub>in</sub> and AIF<sub>out</sub>:  $\sigma^2_{VTF} = AIF_{out}^{(2)} / AIF_{in}^{(0)} - AIF_{in}^{(2)} / AIF_{in}^{(0)} + (AIF_{in}^{(1)} / AIF_{in}^{(0)})^2 - (AIF_{out}^{(1)} / AIF_{out}^{(0)})^2$ , with AIF<sup>(n)</sup> being the n<sup>th</sup> integral moment of the AIF [4]. If dispersion occurs it leads to systematic underestimation of blood flow, which was demonstrated in a simulation study based on the assumption of an exponential VTF [3]. The aim of this study was to simulate the dispersion along a simplified coronary artery with different degrees of stenosis by using the computational fluid dynamics (CFD) approach.

### Material and Methods

Simulations were performed on straight vessels with a length of 10 cm and a diameter of 3 mm having typical dimensions of coronary vessels. Stenoses with different degrees of area reduction (60%, 70%, 80% and 90%) with two different lengths (LoS) of 0.5 cm and 1.0 cm were integrated in the vessels 2cm behind the inlet. The stenoses were symmetric and had a cosinelike shape. The computational meshes consisted of approximately 250,000 quadrilateral cells mapped throughout the domain. The flow equations were solved with the laminar model of a commercial CFD software package (FLUENT, Fluent GmbH, Darmstadt, Germany). The cell-face values were calculated using a second order upwind scheme and the pressure-velocity coupling was achieved by using the SIMPLEC algorithm provided by the software. Two different boundary conditions at the inlet were simulated - to simplify matters both were steady conditions. For the myocardial perfusion at rest a constant plug flow of 0.1 m/s was chosen whereas for perfusion at stress a constant total gauge pressure of 1010 Pa was applied to realize the decrease of the velocity with increasing degree of area reduction of the stenosis. The constant pressure for the stress state was calculated from the results of the vessel without a stenosis with an inlet velocity of 0.5 m/s. A mixture of two species (blood and contrast agent) was simulated in order to derive the transport of the contrast agent. Due to the small mass fraction of the contrast agent (MF<sub>CA</sub>) of less than 0.003 the rheological properties of the contrast agent were neglected. For simplification blood was assumed to be a Newtonian fluid. Therefore, the mixture had a constant density of 1050 kg/m<sup>3</sup> and a dynamic viscosity of 0.04 Poise. The injection of the contrast agent was described by a gammavariate function:  $MF_{CA}(t) = a (t-t_0)^b exp(-(t-t_0) c))$ . The parameters were obtained by fitting the AIF of a volunteer measured in the LV:  $a = 1.013 \ 10^4$ , b = 2.142 and  $c = 1.013 \ 10^4$ . 0.454. The time t<sub>0</sub> which accounts for a delayed arrival of the bolus was set to 0s. The typical duration of a myocardial perfusion measurement of 40s was simulated utilizing a second-order implicit time stepping scheme with a constant time step size of 0.02 s. The area weighted average of the MF<sub>CA</sub> was calculated on several cross sections between the inlet and the outlet perpendicular to the axial vessel direction. To quantify the effect of dispersion, the variance  $\sigma^2_{VTF}$  was calculated for the inlet and each simulated cross section curve.

### **Results**

With a constant inlet pressure the increasing degree of area reduction resulted in a decrease of the inlet velocity with a nearly complete loss of the myocardial perfusion reserve (MPR) for a stenosis of 90% (Fig. 1). On the other hand with a constant inlet velocity the increasing degree of area reduction yielded an increasing pressure drop along the vessel, being slightly higher for the longer stenosis. These are observations expected and were made for checking the validity of the simulations.

The main influence of the stenosis was observed immediately behind it (c.f., Fig. 2). After the initial increase of the variance behind the stenosis a reduction of the variance occurred approximately until the end of the recirculation zone of the velocity field. For a constant inlet velocity at resting condition the effect of the stenosis on the variance at the end of the vessel was only small (Fig 2). At stress condition with a constant inlet pressure, the decrease of the inlet velocity yielded a much stronger dependence of the variance as a function of degree of the stenosis (c.f., Fig.2). The variance for the stress state was clearly less than that for the resting state except for the cases with a stenosis of 90% due to the loss of the myocardial perfusion reserve (MPR) (Fig. 3).



#### Discussion

Under steady boundary conditions dispersion is higher in the resting state than under stress conditions. Dispersion is mainly influenced by the difference in the inlet velocity. By comparison the effect of the stenosis under a constant inlet velocity causes only small increase of the variance of the VTF. The reason for the strong inlet velocity dependence might be the smaller mean vascular transit time. However, this implicates a larger error at quantitation of myocardial blood flow (MBF) in the resting state [3]. In the absence of a stenosis we calculated  $\sigma^2_{VTF} = 6 s^2$  for the resting condition and  $\sigma^2_{VTF} = 0.5 s^2$  under stress conditions. Using an exponential residue function for the VTF and the mathematical model MMID4 Schmitt et al. [3] found for these variance values an error of mean fitted MBF (E<sub>MBF</sub>) of approximately -10% for  $\sigma^2_{VTF} = 0.5 s^2$  and an E<sub>MBF</sub> of approximately -25% for  $\sigma^2_{VTF} = 6 s^2$ . The more severe underestimation of resting MBF leads to an overestimation of the MPR of about 20% for a healthy volunteer with a MPR = 5. For a patient the overestimation would decrease with decreasing MPR.

An implicit assumption of our study is that of steady state conditions, because in this case dispersion is mainly affected by the mean velocity. This is a simplification which needs to be relaxed in future simulations under conditions of pulsatile blood flow and considering the arcuated geometry of coronary vessels. **Acknowledgement** 

# This study was supported in part by the "German Research Council" (grant #Schr 687/1) and by "MAIFOR" and the faculty of Medicine, Johannes Gutenberg-University, Mainz. Further support by the Robert-Müller Foundation, Wiesbaden (Germany) is appreciated.

### References

[1] Jerosch-Herold, M., et al., Med Phys, 1998. 25: p.73-84; [2] Wilke, N., et al., Cardiol Clin, 1998. 16: p.227-46; [3] Schmitt, M. et al., Magn Reson Med, 2002. 47: p.787-93; [4] Calamante, F., et al., Magn Reson Med, 2006. 55: p.1180-5