# Propagation of complex noise in a displacement encoding experiment non-linearly affects quantification of strain

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**OBJECTIVE**: The objectives of this work were twofold: 1) Define a simplified matrix expression that describes the mathematics of an MRI displacement encoding experiment, and 2) Use the simplified expression to characterize the effect of imaging noise on the strain calculated using a Monte-Carlo simulation of displacement encoding with stimulated echoes (DENSE).

**INTRODUCTION**: The DENSE strategy described by Aletras et al has become increasingly utilized for quantitative studies of regional myocardial strains [1]. The existing descriptions of the displacement encoded signal evolution rely upon a either a combination of matrix operations to represent RF pulses, gradient dephasing, and signal crushing combined with vector sums that account for $T_1$ relaxation or they derive similar results using complex analysis. We adopt the approach of Helgstrand [2] and Sorensen [3] who have previously demonstrated the advantages of simplified homogenous coordinate matrix expressions for deriving the MRI signal evolution. The signal acquisition, however, is unavoidably contaminated by noise. Previous work has examined the impact of noise on the displacement measurements [4]. We aim to provide a thorough examination of the impact of noise on the strain measures.

**METHODS**: Homogenous coordinates use 4x4 matrices to effect magnetization vector rotation, dephasing, and relaxation. For the pulse sequence shown in Figure 1 we derive the DENSE signal after the 3rd RF pulse as $DENSE = \mathbf{G}_{\mathrm{a}}^{\eta}\mathbf{R}_{0^o}^{90^o}\mathbf{E}\mathbf{S}\mathbf{R}_{\theta_2}^{90^o}\mathbf{G}_{\mathrm{a}}^{\xi}\mathbf{R}_{0^o}^{90^o}$ where:

$$\mathbf{G}_{\mathrm{a}}^{\xi} = \begin{bmatrix} \mathrm{c}\xi + (1-\mathrm{c}\xi)\,\mathrm{a}_1^2 & (1-\mathrm{c}\xi)\,\mathrm{a}_1\mathrm{a}_2 - \mathrm{s}\xi\mathrm{a}_3 & (1-\mathrm{c}\xi)\,\mathrm{a}_1\mathrm{a}_3 + \mathrm{s}\xi\mathrm{a}_2 & 0 \\ (1-\mathrm{c}\xi)\,\mathrm{a}_1\mathrm{a}_2 + \mathrm{s}\xi\mathrm{a}_3 & \mathrm{c}\xi + (1-\mathrm{c}\xi)\,\mathrm{a}_2^2 & (1-\mathrm{c}\xi)\,\mathrm{a}_2\mathrm{a}_3 - \mathrm{s}\xi\mathrm{a}_1 & 0 \\ (1-\mathrm{c}\xi)\,\mathrm{a}_1\mathrm{a}_3 - \mathrm{s}\xi\mathrm{a}_2 & (1-\mathrm{c}\xi)\,\mathrm{a}_2\mathrm{a}_3 + \mathrm{s}\xi\mathrm{a}_1 & \mathrm{c}\xi + (1-\mathrm{c}\xi)\,\mathrm{a}_3^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{R}_{\theta}^{\alpha} = \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}\theta\mathrm{s}\alpha & 0 \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & 0 \\ -\mathrm{s}\theta\mathrm{s}\alpha & \mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{E} = \begin{bmatrix} E_2 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_1 & m_0\,(1-E_1) \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**G** is the gradient dephasing matrix, **R** is the RF-pulse matrix, **E** is the relaxation matrix, and **S** is the spoiling matrix. Where, "c" and "s" represent cosine and sine functions, $a_i$ are the normalized vector components for the direction of the applied gradient, $\xi$ is the gradient imparted phase and $\eta$ is the phase unwinding. $\theta$ and $\alpha$ are the RF phase and flip angle, $E_1$=1-exp(-t/$T_1$), $E_2$= exp(-t/$T_2$), and $m_0$ is the magnitude of the equilibrium magnetization. Note the order of matrix operations is left-right reversed with respect to Fig. 1.

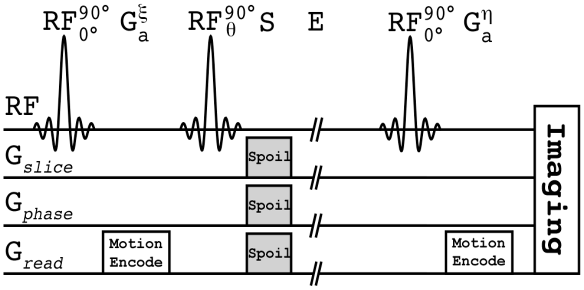

**FIGURE 1.** DENSE pulse sequence diagram.

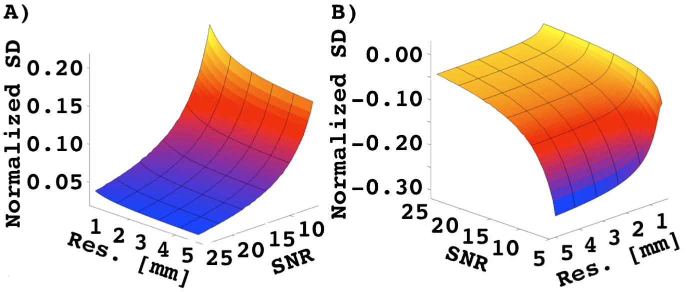

**FIGURE 2.** Normalized strain error for (A) stretch and (B) compression.

Monte-Carlo simulations with $128^2$ repetions were performed for a range of image resolutions (0.5mm to 5mm) and SNR (5 to 25), and for 1-D extension ($\lambda$=1.2, 22% strain) and compression ($\lambda$=0.75, -22% strain) by adding Gaussian random noise to each of the simulated signals for a 3-point DENSE experiment (RF phases were 0°, 120°, and 240° and 0.86mm/$\pi$ encoding strength). The tissue $T_1$ was 650ms and the delay between the 2nd and 3rd RF pulses was 300ms. The strain error is presented as the standard deviation of the noisy strain estimates normalized by the true strain value.

**RESULTS**: Figure 2 demonstrates that the strain error is non-linearly dependent on the image SNR and the image resolution (Note: axes are rotated for best view). The non-linear effect is related to both the underlying MRI physics and the equation for calculating the Lagrangian strain. This result highlights the fact that careful selection of the imaging resolution is important for minimizing the error in the strain measurement. Note, however, that for image SNR>~15 the normalized strain standard deviation is low and relatively independent of the image resolution.

**DISCUSSION**: The advantage of the homogeneous coordinate expression for the DENSE signal is twofold: 1) The matrix expression is easy to interpret as it directly correlates to the RF and gradient events in the pulse sequence diagram, and 2) the matrix formalism facilitates evaluation of the noise analysis especially when using programming languages that are matrix "aware" (eg Matlab or Octave). The simulation demonstrates that it is important to carefully consider the imaging resolution used in a DENSE experiment. Increases in imaging resolution (decreasing pixel size) can increase the error in the strain estimate even when SNR is held constant.

**REFERENCES**: [1] Aletras AH, JMR 1999;137(1):247-252; [2] Helgstrand M, J Bio. NMR 2000;18(1):49-63; [3] Sorensen OW, NMR Spect. 1983; 16:163–192; [4] Aletras AH, JMR 1999;140(1):41-57.

**ACKNOWLEDGEMENTS**: DBE supported by NIH K99-HL087614.