Non-Fourier Imaging and Fast B0 Mapping with Linearly Ramped Gradients

L. Sacolick[1], D. L. Rothman[2], and R. A. de Graaf[2]

[1]Biomedical Engineering, Yale University, New Haven, CT, United States, [2]Diagnostic Radiology, Yale University, New Haven, CT, United States

INTRODUCTION: The overwhelming majority of current applications in NMR imaging use Fourier-based spatial localization by constant phase and read gradients. It is possible to encode spatial information in the time dimension during acquisition with time-varying gradient functions, however the image can no longer be directly reconstructed by Fourier transform (1,2). For the large majority of time-encoded methods such as spiral and radial imaging and SPRITE, images are reconstructed by regridding k-space, effectively removing the time-dependence of the signal in favor of frequency encoding. In principle, spins can be uniquely spatially encoded by a variety of approaches besides k-space based frequency encoding, and these can have significant advantages and applications in NMR. Here an algorithm will be presented for a non-Fourier vector-space projection method for directly reconstructing images acquired with arbitrary gradient shapes. The examples here will be demonstrated with linearly increasing ramped gradients although the technique is not at all limited to these gradient functions. One potential application of the technique, fast single-image B_0 mapping, will be demonstrated here.

THEORY: Consider a read gradient along the Cartesian *x*-dimension, increasing linearly at rate *A* (Hz/cm/s) with gradient function $G_x(t)= G_x(0)+At$ during acquisition. For simplicity $G_x(0)$: the amplitude of the read-gradient at the start of acquisition will be disregarded here. In the case of a ramp gradient the frequency of a spin at position *x* with constant offset frequency $\omega(x)$ will vary linearly with time as $\omega(x)+Axt$. The signal $s(t)$ acquired during this acquisition is given by: $s(t)=\int \rho(x)e^{-i\pi Axt^2+\omega(x)t}dx$, where $\rho(x)$ is the spin density along a projection in the *x*-direction. The rate of the change of frequency Ax depends on position in the sample and can be used for spatial encoding along *x*. Ramp gradient encoding that distinguishes spatial position based on the linear gradient frequency slope Ax would be insensitive to a large range of B_0 inhomogeneity.

The functions $\psi(x,\omega,t)=e^{-i\pi Axt^2+\omega(x)t}$ are examples of *linear chirp* functions. For all values of *x* and ω (such that the largest possible frequency of $Ax+\omega(x)$ falls within the Nyquist sampling constraint), these chirp vectors are unique, linearly independent, and orthogonal in the sense that the autocorrelation of each of these signals approximates a delta function with low side-lobe amplitude (3). This allows the simultaneous detection of the spin density along *x*, $\rho(x)$, and the B_0 offset frequency, $\omega(x)$.

This method is easily extended to 3D ramp gradient-echo imaging with one ramp-encoded dimension as described above, one conventionally phase-encoded, and one conventionally slice-encoded dimension. The phase-encoded dimension can be resolved by 1-D Fourier transform. The *x* and ω dimensions are resolved by vector-space projection where the basis functions consist of vectors $\psi(x,\omega,t)=e^{-i\pi Axt^2+\omega(x)t}$ where *x* spans positions in the sample and ω spans the range of B_0 inhomogeneity found over the sample. The inner product of the signal $s(t)$ with each basis vector $\psi(x,\omega,t)$ that corresponds to position *x* and offset ω gives the spin density at each position, at each B_0 offset. The result is a spectroscopic image for each pixel in the slice. To get a B_0 map, the maximum of the frequency spectrum is calculated for each pixel. To get an image, the spectroscopic image is integrated over the frequency dimension for the sum of the spin density at all frequencies for each pixel- in effect giving spatial localization based only on the gradient slope.

METHODS: The ramp gradient-echo sequence shown below was implemented on a 4.0 T Magnex magnet interfaced to a Bruker Avance spectrometer. RF reception and transmission were carried out by a Bruker birdcage volume coil (ID = 26 cm). Ramp gradient-echo images and B_0 field maps were acquired for a saline phantom and a human brain. For comparison with conventional techniques, these were compared to gradient-echo MRI images and gradient echo B_0 field maps with temporal phase unwrapping. The ramp gradient images were oversampled during acquisition with 1024 time points x 128 phase encoding steps acquired to reconstruct a 128x128 image and a 128x128 field map.

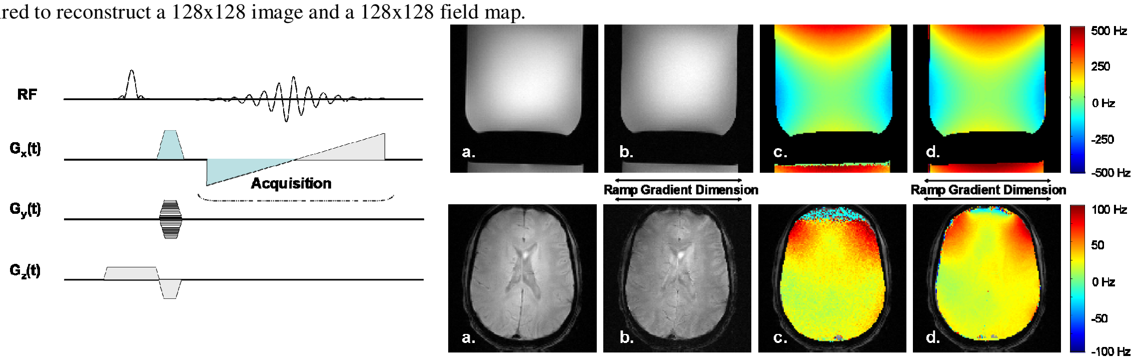

Figure 1: Left: ramp gradient-echo sequence used for both image and B_0 field map acquisition. **(a.)** Conventional gradient-echo MRI of an unshimmed cylindrical saline phantom (top) and human brain (bottom). **(b.)** Ramp gradient-echo MRI, 1024 acquisition time points, gradient ramp rate A=1006 kHz/cm/s. **(c.)** Conventional 128x128 B_0 field maps calculated from the unwrapped phase of four gradient-echo images with varying free-precession delays. **(d.)** B_0 maps calculated from the single ramp gradient echo images shown in **b.** The B_0 map was reconstructed for 128 points over a range of ±400 Hz in the frequency dimension ω. All four sets of images were acquired with TR = 500 ms, 100 kHz sampling rate, FOV = 19.2x19.2 cm, resolution = 128x128, slice thickness = 0.5 cm. The conventional gradient echo images and B_0 maps were acquired with a TE of 12 ms, and the ramp-gradient echo images were acquired with TE = 17 ms.

RESULTS AND DISCUSSION: A read-gradient that is linearly ramped during acquisition produces NMR signals in the form of complex linear chirp functions. From these signals it is possible to reconstruct both the spatial location and frequency offset of a spin. Since the spatial position along the ramp-direction is encoded by the gradient slope and not frequency, spatial encoding in that dimension is insensitive to B_0 inhomogeneity. This allows the reconstruction of a B_0 map from a ramp gradient-echo image. A field map can be calculated from a single ramp gradient-echo image by this method far more quickly than any of the conventional multiple phase image techniques. Although demonstrated here for a linear ramp gradient shape, this technique is not limited to any specific gradient shape or limited to shaped gradients in only one dimension. The vector-space projection can be applied to a wide variety of functions: sinusoidal, spiral, trapezoidal, multiple ramp gradient functions, etc. However a key assumption of this technique is that the vectors in ψ form an orthonormal basis set.

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