Optimizing k-t BLAST/SENSE using FOCUSS and RIGR

H. Jung[1], and J. C. Ye[1]

[1]Bio and Brain engineering, Korea Advanced Institute of Science & Technology (KAIST), Guseong-dong Yuseong-gu, Daejon, Korea, Republic of

Introduction: Recently, compressed sensing theory [1] has been an active area of research in MR community due to its potential for high resolution reconstruction even from very limited number of samples even below the Nyquist sampling. In [2], we developed a new dynamic MRI algorithm called k-t FOCUSS (k-t space FOCal Underdetermined System Solver) that is optimal from compressed sensing point of view, and showed that the existing k-t BLAST/SENSE is an approximation of our k-t FOCUSS algorithm. The main contribution of this paper is to show that our k-t FOCUSS algorithm can be further improved by incorporating RIGR (Reduced-encoding Imaging by Generalized-series Reconstruction)[3] initialization and the recently proposed SPEAR (SPatiotemporal domain based unaliasing employing sensitivity Encoding and Adaptive Regularization) [4] algorithm is an approximation of our algorithm. Experimental results confirm our theory.

Theory: According to the compressed sensing theory, accurate reconstruction can be achieved by solving L1 minimization problem when the unknown image is sparse in some bases. In [1], we applied FOCUSS (FOCal Underdetermined System Solver) as a solution technique for L1 minimization technique in cardiac cine imaging. Our k-t FOCUSS algorithm obtains the sparse solution in x-f domain by iteratively solving weighted L2 minimization problem in k-t space. Interestingly, the celebrated k-t BLAST/SENSE turns out to be the first iteration of our k-t FOCUSS that is asymptotically optimal from compressed sensing perspective. Specifically, k-t FOCUSS acquires fully sampled low frequency data and randomly sampled high frequency data in k-t space. Let ρ be the unknown sparse signal to be reconstructed. Then, we can decompose ρ by two terms; initialization, ρ_0, and the residual $\Delta\rho$ which should be reconstructed through iterative process. If v , W , and F represent the k-t measurements, weighting matrix, and sparsifying transform, respectively, k-t FOCUSS can be briefly summarized by following equation.

$$\text{Find } \Delta\rho_{n+1} = W_n q_n$$
$$\min \ \| q_n \|_2 \ , \text{ subject to } \ \| v - F\rho_0 - FW_n q_n \|_2 \le \varepsilon \tag{1}$$

Then, until convergence, W_n is updated using Eq. (2)

$$W_n = diag(abs(\Delta\rho_n)^p) \ , \ \ 1/2 \le p \le 1 \tag{2}$$

Here, setting $p = 0.5$, it can be easily shown that k-t FOCUSS asymptotically solves L1 minimization problem for $\Delta\rho$. In [2], a temporal average was used as the initialization ρ_0,. Even though this choice still provided excellent reconstruction, this is by no means optimal in the sense that quite significant energy could remain in the residual signal $\Delta\rho$ due to the signal nulling problem. In this paper, we propose a RIGR method [3] to set initialization term, ρ_0. Basically, RIGR assumes that significant features for the dynamic information are usually in low frequency region and RIGR reconstruct dynamic images from only small number of low frequency PEs and a couple of reference frames [3]. Since k-t FOCUSS is already acquiring fully sampled low frequency data in k-t space, RIGR can be easily incorporated by adding just two additional reference frames at before and after acquiring dynamic encodings. The main advantage of incorporating RIGR as ρ_0 is that the unknown residual signal $\Delta\rho$ can be significantly reduced by eliminating the signal nulling problem; hence, we just need much smaller number of k-t samples compared to the temporal average initialization . Specifically, the cost function Eq. (1) can be solved iteratively with updated weighting matrix $\Theta_n = W_n W_n^H$ at each iteration :

$$\rho = \rho_0 + \Theta_n F^H (F\Theta_n F^H + \lambda I)^{-1}(v - F\rho_0) \qquad \text{where } \ \Theta_n = W_n W_n^H \tag{3}$$

Interestingly, the first iteration of Eq. (3) is equivalent to recently proposed SPEAR algorithm [4], which sets ρ_0 term with RIGR instead of temporal average in the conventional k-t BLAST/SENSE. Even though SPEAR can remove the signal nulling problem of k-t BLAST/SENSE, it is not optimal from compressed sensing point of view. However, our k-t FOCUSS simply incorporates more iterations, which makes the algorithm asymptotically optimal from compressed sensing perspective. Hence, k-t FOCUSS is very useful both in theory and actual applications.

Result: To validate our theory, we compared the results of k-t FOCUSS with those of k-t BLAST and SPEAR for 6.5 fold accelerated cardiac cine data. In order to show the improvements clearly, difference images between reconstruction and fully sampled image are also illustrated in Fig. 1. Here, k-t FOCUSS effectively removes aliasing artifacts, whereas the other methods still exhibit the remaining aliasing artifacts marked by white arrow.

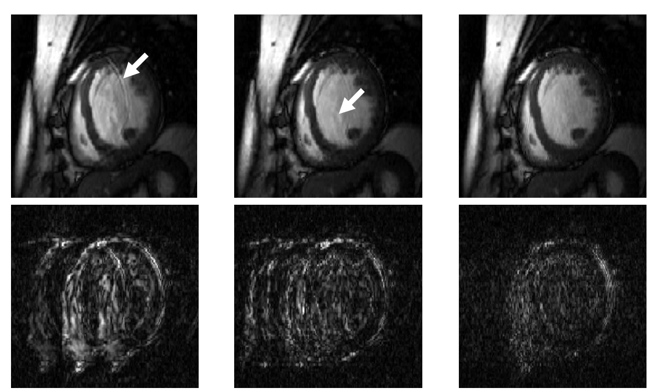

Fig.1 (a) k-t BLAST (b) SPEAR (c) k-t FOCUSS

Conclusion: By applying FOCUSS iteration in k-t space with RIGR initialization, we can significantly improve the reconstruction quality of dynamic cardiac cine imaging. The main advantage of RIGR initialization is to effectively eliminate the signal null problem often observed in k-t BLAST/SENSE. Furthermore, FOCUSS iteration effectively eliminates the aliasing artifacts by exploiting the sparsity of the residual signals. Our k-t FOCUSS with RIGR initialization is a generalization of k-t BLAST/SENSE as well as recently proposed SPEAR algorithm, and k-t FOCUSS is asymptotically optimal from compressed sensing point of view.

References

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