

# Image Reconstruction from Ambiguous PatLoc-Encoded MR Data

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**Introduction:** Recently, the PatLoc concept of spatial encoding using non-bijective nonlinear magnetic fields has been proposed [1]. It has several advantages over traditional linear gradient encoding, particularly to overcome peripheral nerve stimulation limitations in special applications such as cortical imaging. Another advantage of PatLoc is that such fields can be tailored specifically to the anatomical structures being imaged. We present here the first practical reconstruction approach capable of image restoration encoded with curvilinear non-bijective fields. Unlike the previously reported methods [2], our present approach explicitly considers overlapping receiver coil sensitivities and applies a generalization of the SENSE technique [3] to resolve ambiguities of the PatLoc encoded MR data.

**Theory:** For the sake of demonstration a perfect radial symmetry of the encoding fields is assumed and a single excited slice is considered. Fig. 1 shows as example six-pole encoding fields, each having three rotated bijective regions. The fields define a function  $\vec{B}(x, y) = (B_x(x, y)/\Delta B_x, B_y(x, y)/\Delta B_y)^T$ , where  $\Delta B_x$  and  $\Delta B_y$  describe the difference between minimum and maximum value of the encoding fields. It is useful to adopt the k-space concept to arbitrary encoding fields by defining  $k_x = \gamma \Delta t \Delta B_x$  and  $k_y = \gamma \Delta t \Delta B_y$ , where  $\Delta t$  is the readout sampling time and  $T$  is the duration of the phase-encoding field pulse. With the substitution rule for multiple variables, the signal of a receiver coil  $\alpha$  with sensitivity  $C_\alpha$  is given by:

$$S_\alpha(\vec{k}) = \int_V M(\vec{r}) C_\alpha(\vec{r}) \exp(-i\vec{k}\vec{B}(\vec{r})) d^2r = \int_U \left( \sum_{i=1}^n \tilde{M}_i(\vec{s}) \tilde{C}_{\alpha,i}(\vec{s}) d_i(\vec{s}) \right) \exp(-i\vec{k}\vec{s}) d^2s, \quad (1)$$

where  $U = \vec{B}(V_i)$  with  $\{V_i \subset V, i=1, \dots, n\}$  being a partition of  $V$  and  $\vec{B}$  being bijective on each region  $V_i$  with  $i=1, \dots, n$  and  $\vec{s} = \vec{B}(\vec{r})$ . The transformation from the object space to the encoding space is not linear as in traditional Fourier imaging, so that a volumetric correction factor  $d_i$  must be considered explicitly. It is given by  $d_i(\vec{s}) = 1/|\det(\partial \vec{B}(\vec{r}_i)/\partial \vec{r}_i)|$ . This transformation makes Eq. (1) accessible to FFT. The encoding process can then be written as a matrix equation depending on  $\vec{s}$ , which can be solved by a matrix inversion:

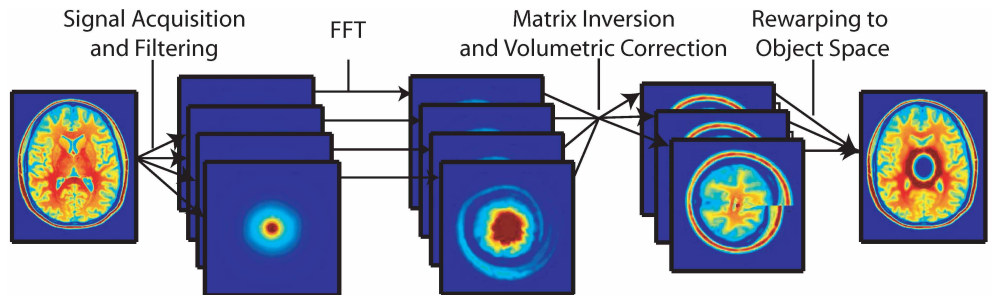
$$M = D(C^{-1}S). \quad (2)$$

$S$  and  $M$  are vectors containing the signals and magnetizations,  $C$  contains the sensitivity information and  $D$  is a diagonal matrix containing the inverse values of the volumetric correction factors.  $C$  and  $D$  are assumed to be invertible.

**Methods:** Starting from a spin density (e.g. with data as shown in Fig. 3, left) the signal was simulated with the encoding fields shown in Fig. 1. A rectilinear trajectory in k-space was used to sample the signals. The sensitivity information for each receiver coil was chosen to be linearly independent at each point such that the matrix  $C$  is invertible. Prior to FFT of each receiver channel signal, a properly tuned Kaiser-Bessel filter [4] was applied. Thereafter Eq. (2) was solved for each point in the encoding space. Fig. 4 outlines the reconstruction algorithm.

**Results:** Fig. 2 and Fig. 3 illustrate reconstruction results of a random noise phantom and a brain "phantom" in comparison to Fourier imaging. A very good image quality can be appreciated at the periphery of the imaged area, while the resolution decreases gradually towards the center of the image, caused by the fact that the gradients of the magnetic encoding fields flatten around the center of the PatLoc coil. Additional loss of resolution comes from the filtering operation, which at present appears unavoidable. The reconstruction algorithm has recently been tested with the encoding fields generated by the real PatLoc gradient coil. It has proven to be robust against relatively strong perturbations destroying the ideal symmetry of the encoding fields. The choice of encoding fields as in Fig. 1 has been motivated by two reasons from a reconstruction point of view. First, the encoding fields are mutually orthogonal, leading as in Fourier imaging to better imaging properties. Second, the volumetric correction represented by the matrix  $D$  is independent of the polar angle and  $D$  is invertible for each point.

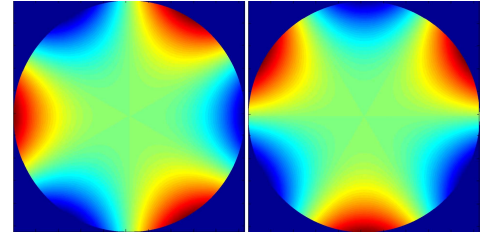
**Discussion:** Presented here is the first efficient and practical reconstruction method for general non-bijective and nonlinear encoding fields. The method shares many properties with traditional parallel imaging techniques, therefore also many advantages of parallel imaging are transferable to PatLoc imaging. However, in contrast to SENSE or GRAPPA, acceleration in the PatLoc reconstruction is not determined by the omitted k-space lines, but rather by the number of bijective encoding regions. In spite of the different background to SENSE imaging, the problem of resolving ambiguities leads to a generalized SENSE-like method, which treats the nonlinearities not as perturbation, but as integral part of the reconstruction algorithm. This leads to a nonlinear relation between the object space and the encoding space and many global invariants such as FoV, PSF or resolution become locally dependent. It seems feasible to generalize the PatLoc encoding to allow for the traditional parallel acceleration, and, depending on the k-space sampling strategy, this will probably require revisiting of the iterative SENSE approaches [4]. The algorithm is in principle capable of dealing with any nonbijective, nonlinear encoding fields. The real-life test of the algorithm with the data measured using the PatLoc coil is yet to follow in the near future.



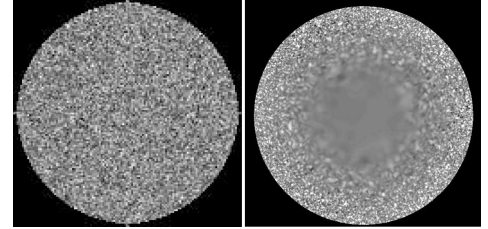
**Figure 4:** Outline of the reconstruction algorithm. First, the simulated or measured data are filtered; second, a FFT is applied; third, a matrix inversion is performed for each point and volumetric correction is applied; at last, the images are rewarped to object space.

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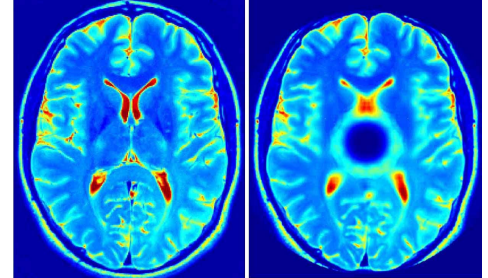
**References:** [1] Hennig et al., ISMRM, 2007, 453; [2] Hennig et al., submitted to MAGMA; [3] Pruessmann et al., MRM 42, 952-962, 1999; [4] Harris, Proc. IEEE 66, 51-83, 1978; [5] Pruessmann et al., MRM 46, 638-651.



**Figure 1:** Rotated spatial encoding fields with six poles.



**Figure 2:** Random noise phantom. Left: Fourier reconstruction. Right: PatLoc reconstruction. Signals were simulated with  $128 \times 128$  k-space points.



**Figure 3:** Brain "phantom". Left: Fourier reconstruction. Right: PatLoc reconstruction. Signals were simulated with  $256 \times 256$  k-space points.