Optimal Control Design of Phase-Relaxed Parallel Transmission RF Pulses for Arbitrary Flip Angles

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INTRODUCTION

Conventional parallel transmission RF pulse designs [1-3] are limited in two aspects: 1) an overly restrictive flat phase profile of the desired magnetization is enforced, which may sacrifice the quality of the magnitude profile, and 2) only small-tip-angle (STA) pulses can be accurately designed. Within the STA regime, the first issue has been addressed by the phase-relaxed design with a predetermined phase [4], which can be further improved by the optimal phase-relaxed STA design (submitted to ISMRM 08). To address the second issue, several methods have been proposed to design large-tip-angle or arbitrary-tip-angle parallel transmission pulses, which include the optimal control design [5,6]. However, none of the methods addresses both issues. In this paper, we propose a new, spinor-based, optimal control method which is capable of designing parallel transmission pulses with optimal phase profile and arbitrary tip angle simultaneously.

PROPOSED METHOD

Denote $\boldsymbol{\alpha}(t)$ and $\boldsymbol{\beta}(t)$ as vectors that vertically concatenate spatial domain samples of the corresponding Cayley-Klein parameters in the spinor form Bloch equation [7], where t represents the time axis, $\boldsymbol{\alpha}_{\text{des}}$ and $\boldsymbol{\beta}_{\text{des}}$ as the desired Cayley-Klein vectors at the terminal time T, and $b_l(t)$ as the RF pulse of the lth coil, $l = 1,2,\dots,L$. The phase-relaxed, spinor based, optimal control pulse design is formulated as:

$$\text{Choose } b_1(t),\dots,b_L(t) \text{ to minimize } J(b_1,\dots,b_L)=\|\boldsymbol{\alpha}(T)-\boldsymbol{\alpha}_{\text{des}}\|^2_{\mathbf{W}_1}+\||\boldsymbol{\beta}(T)|-\boldsymbol{\beta}_{\text{des}}\|^2_{\mathbf{W}_2}+\lambda\sum_{l=1}^{L}\int_0^T|b_l(t)|^2\,dt, \quad (1)$$

where $\boldsymbol{\alpha}(t)$ and $\boldsymbol{\beta}(t)$ follow the Bloch equation, $\mathbf{W}_1$ and $\mathbf{W}_2$ are diagonal matrices with their diagonal elements representing spatial weights, $\|\cdot\|^2_{\mathbf{W}_1}$ and $\|\cdot\|^2_{\mathbf{W}_2}$ are weighted l_2 norms, and $|\cdot|$ denotes the magnitude operator. The first and second terms in Eq. (1) penalize the overall error in final magnetization and the third term approximately penalizes the RF power consumption where λ is a regularization parameter. There are a number of advantages of formulating RF pulse design using Eq. (1): Firstly, this formulation is capable of designing high quality RF pulses with arbitrary tip angles because the Bloch equation nonlinearity is directly addressed. Secondly, solving Eq. (1) can be faster than solving the magnetization vector-based formulation [5] because of the reduced matrix size and capability of seamlessly using spinor-based Bloch equation calculation [8]. Thirdly and most importantly, based on the definition of $\boldsymbol{\beta}(t)$ [7], as shown in Eq. (1), removing the phase of transverse magnetization is simply a magnitude operation on $\boldsymbol{\beta}(T)$ (which permits a closed form gradient vector of J to enable efficient numeric algorithm) while for the magnetization vector-based formulation, the magnitude operation mixes components of the magnetization vectors, which can complicate the design. Note we have assumed the gradient is refocused (i.e., the integral of gradient waveform is zero) and therefore $\boldsymbol{\alpha}(T)$ and $\boldsymbol{\alpha}_{\text{des}}$ are real.

To solve Eq. (1), we first rewrite the Bloch equation as an explicit form of real and imaginary parts of $b_l(t)$ weighted by precomputed matrices (which include transmit sensitivity and field gradient terms, and details are omitted for limited space). Then we introduce two co-state vectors $\boldsymbol{\sigma}(t)$ (corresponding to $\boldsymbol{\alpha}$) and $\boldsymbol{\tau}(t)$ (corresponding to $\boldsymbol{\beta}$) to convert the constrained problem into an unconstrained one. Based on the unconstrained formulation, we enforce the first order necessary conditions for the optimal solution, which yield a set of differential equations for $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\tau}$ with the following boundary conditions:

$$\boldsymbol{\alpha}(0)=[1,1,\dots,1]^{\mathrm{T}},\ \boldsymbol{\beta}(0)=[0,0,\dots,0]^{\mathrm{T}},\ \boldsymbol{\sigma}(T)=\mathbf{W}_1(\boldsymbol{\alpha}(T)-\boldsymbol{\alpha}_{\text{des}}),\ \boldsymbol{\tau}(T)=\mathbf{U}\boldsymbol{\beta}(T),\ \text{where } \mathbf{U}=\operatorname{diag}\{w_{2,m}-\beta^2_{\text{des},m}w^2_{2,m}/\sqrt{\beta_{\text{des},m}w_{2,m}|\beta_m(T)|^2+\delta}\}, \quad (2)$$

$w_{2,m}$ is the mth diagonal element of $\mathbf{W}_2$, $\beta_{\text{des},m}$ is the mth element of $\boldsymbol{\beta}_{\text{des}}$, $\beta_m(T)$ is the mth element of $\boldsymbol{\beta}(T)$, and δ is a small constant introduced to overcome the nondifferentiability of the magnitude operation at the origin [9]. Similar to the phase enforced design [5,6], the differential equations with boundary conditions in Eq. (2) form a two-point boundary value problem, which does not have closed form solution. We solve the problem using a first order gradient algorithm similar to [5,6].

RESULTS

Based on simulation of dual-channel transmission of 90° RF excitation pulses for B_1 inhomogeneity correction (where transmit sensitivity maps are acquired with a torso phantom), four different pulse designs are compared in Fig. 1. The STA design with a flat phase (conventional design) has under-flipped magnetization with large spatial variation due to the Bloch equation nonlinearity at 90° and the overly restrictive flat phase constraint. STA with optimal phase (in another abstract submitted to ISMRM 08) reduces the spatial variation by relaxing the phase constraint, but still has an overall under-flipped magnetization especially near the center due to the Bloch equation nonlinearity. Optimal control design with flat phase [5] addresses the Bloch equation nonlinearity and therefore generates the correct flip angle for most locations. However, some spatial variations in the magnitude are still visible. The optimal control design with optimal phase further improves the homogeneity of the magnitude profile by relaxing the phase constraint. Comparison of 1D magnitude profile is also shown in Fig. 1 to reveal details. The average magnitudes of M_{xy} for the four methods are 0.78, 0.81, 0.98, and 0.99, respectively, and the standard deviations are 0.13, 0.052, 0.040, and 0.016, respectively.

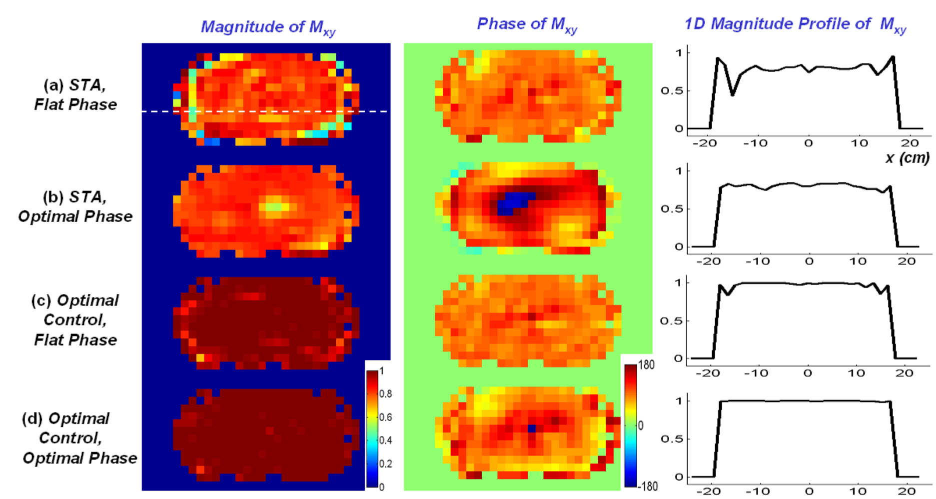

Fig. 1. Bloch simulation results comparing M_{xy} of 90º dual-channel RF excitation pulses from four pulse designs for B_1 inhomogeneity correction: (a) STA design with flat phase, (b) STA design with optimal phase, (c) optimal control design with flat phase, and (d) optimal control with optimal phase. The first three designs suffer either from Bloch equation nonlinearity (a and b) and/or overly restrictive flat phase constraint (a and c), therefore producing under-flipped magnetization and/or spatial variation in the magnitude profile. All magnetizations are normalized by the equilibrium magnetization constant.

CONCLUSION

The optimal control design with optimal phase profile relaxes the phase constraint of the desired magnetization and addresses the Bloch equation nonlinearity simultaneously, and can therefore be used to design arbitrary-tip-angle parallel transmission RF pulses with improved accuracy of the magnitude profile over conventional methods. In addition to the 90° excitation pulse example, the proposed design should also be applicable to the design of inversion and refocusing pulses.

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