Optimal Sample Parameter Estimates from Phased Array Coil Data Utilizing Joint Bayesian Analysis

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Numerous investigators, e.g. (1-3), have proposed techniques for combining data from phased array coils to produce the optimal image SNR. Typically, a type of sensitivity-weighted average is used, though the sum-of-squares (SOS) combination is advocated as an easily implemented approximation to this "optimal" result. One aspect that has received relatively little attention is the optimal coil combination technique if a series of such images are being utilized to produce parametric estimates. Herein we address this question by utilizing Bayesian probability theory to jointly analyze the data from multiple channels, allowing each channel to have independent amplitudes and noise powers. We have also compared the joint analysis to the standard alternatives to determine the technique that produces the most precise parameter estimates. Our results demonstrate that a joint Bayesian analysis offers a "worry free" method for obtaining optimal parameter estimates from the analysis of array-coil data.

Theory: For a simple mono-exponential model, the signal in the m-th coil at the imaging time t_n is modeled as $S_m(t_n) = A_m \cdot \exp(-R\,t_n)$. The joint posterior probability of the coil amplitudes $A = \{A_m\}$ and relaxation rate constant R given the data, D, the standard deviation of the noise for each channel, $\sigma = \{\sigma_m\}$, and the prior information, I, is given by (4):

$$p(A,R|D,\sigma,I) \propto \exp\left(-\frac{1}{2}\sum_{m=1}^{M} Q_m/\sigma_m^2\right),\quad Q_m = \sum_{n=0}^{N-1}\left[S_m(t_n) - D_m(t_n)\right]^2, \tag{1}$$

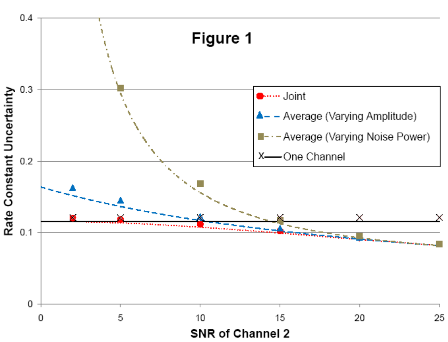
where $D_m(t_n)$ is the observed data on channel m at time t_n. The uncertainty in the estimate of R, σ_R, can be evaluated by integrating $p(A,R|D,\sigma,I)$ over all possible values of the amplitudes and utilizing the Laplace approximation (5) . The result is as follows:

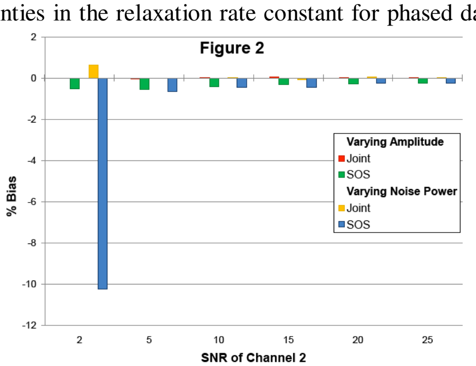
$$\sigma_R = (R/SNR_{Joint})\cdot F(R,t_n),\qquad F(R,t_n) = \left[\Sigma_0/\left(\Sigma_0\Sigma_2 - \Sigma_1^2\right)\right]^{1/2},\quad \Sigma_i = \sum_{n=0}^{N-1}(R\,t_n)^i \exp(-2R\,t_n),\quad SNR_{Joint} = \left(\sum_{m=1}^{M} SNR_m^2\right)^{1/2}. \tag{2}$$

where $SNR_m = A_m/\sigma_m$ is the SNR in the m-th coil. For weighted averaging (WAv), $D(t_n) = \sum \lambda_m D_m(t_n)$ and Eq. [2] is modified by the substitution: $SNR_{Joint} \to SNR_{WAv} = \sum \lambda_m A_m / \left(\sum \lambda_m^2 \sigma_m^2\right)^{1/2}$. For an un-weighted average, $\lambda_m = 1$; for sensitivity weighting, $\lambda_m = A_m$. Using the Cauchy inequality, it can be proven that $SNR_{Joint} \geq SNR_{WAv}$ and therefore $\sigma_R^{Joint} \leq \sigma_R^{WAv}$; with equality at $\lambda_m = A_m/\sigma_m^2$.

Methods: These theoretical predictions were verified by analyzing simulated two-channel relaxation data (160,000 sets) with normally-distributed noise which were generated using each of the following parameter sets: $A_1 = 25$, $\sigma_1 = 1$, R=1, t_n = {0, 1}, and SNR_2 = {25, 20, 15, 10, 5, 2}; where the decrease in SNR_2 was generated by either decreasing A_2 or increasing σ_2 from the channel 1 values. The multichannel data was analyzed jointly, using the high SNR channel alone, or by using the average and SOS combination of channels. Estimates for the relaxation rate constant and its uncertainty were calculated from the posterior probability distribution function in equation (1) for each data set, combination, and parameter set using customized MATLAB routines. A second series of data were generated using magnitude data for each channel and analyzed as above.

Results: The theoretically predicted uncertainties in the relaxation rate constant for phased data as a function of SNR_2 are shown as lines in Fig. 1 and the average uncertainties over the simulations are shown as data points. The data show excellent agreement with the theoretical values except in cases of extremely low SNR_2. As the SNR of the second channel decreases from SNR_1 to zero, the uncertainty in R from the joint analysis transitions from $\sqrt{2}$ lower than the one channel result to the one channel result. In contrast, while the un-weighted average has the same uncertainty when both channels are equal, it will produce substantially higher uncertainties than even the single channel result when the signals on the two channels are significantly different, especially if that difference is due to an increase in the noise power.





The uncertainty from the sensitivity-weighted average ($\lambda_m = A_m$) and SOS are omitted for clarity as they would overlap the joint analysis when the amplitude of the second channel is changing and the average analysis when the noise power is changing. Thus, while the joint analysis is either improved or unaffected by the inclusion of low SNR channels, parameter estimates can be significantly corrupted from alternate combination methods that do not properly account for differences in channel amplitudes and noise powers. Uncertainties obtained from magnitude data show similar patterns.

Biases in the relaxation rate constant estimates from phased data are shown in Fig. 2. The SOS combination results in increased bias over joint analysis and this effect increases if the noise power varies between channels or as any of the data values approach the noise floor. While bias is always present using magnitude data, the bias from the joint analysis is always smaller than or comparable to the bias from the SOS combination.

Conclusion: Our results demonstrate that joint Bayesian analysis offers a "worry free" method for obtaining optimal parameter estimates from the analysis of array coil data. While combining channels weighted by $\lambda_m = A_m/\sigma_m^2$ can produce optimal estimates (within the accuracy that this factor is known), the use of incorrect weighting factors corrupts the parameter estimates. This effect is amplified for low SNR, increased number of low SNR channels, increased number of low SNR points in the relaxation curve, variations in noise power across channels, and with magnitude data.

References: **1**. Roemer PB. MRM 1990;16(2):192-225. **2**. Walsh DO. MRM 2000;43(5):682-690. **3**. Larsson EG. JMR 2003;163(1):121-123. **4**. Bretthorst GL: Springer-Verlag; 1988. 210 pp. **5**. Bretthorst GL. Concepts Magn Reson 2005;27A(2):73-83.