Using Boy's Real Projective Plane Immersion for Coloring DT-MRI Slices

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Introduction:
We introduce Boy's surface, an immersion of the projective plane (RP2) in 3D, as a model for mapping tensor orientation to colors. One of the most popular methods of visualizing DT-MRI slices is to map principal eigenvectors of the underlying tensors to RGB colors. An extensive study of different coloring methods for visualizing DT-MRI can be found in [1]. All the existing methods, however, suffer from mirror symmetry one way or another. Ideally, we would like to map 1) different orientations to different, unique colors and 2) similar orientations to similar colors. As trivial as it sounds, this task is not easy and there is a compelling reason for that: The problem of smooth, one-to-one assignment of colors to orientations is tantamount to embedding the real projective plane in 3D, which was proven to be impossible. While the real projective plane does not have an embedding in 3D, it has immersions. Here, we present Boy's immersion of the real projective plane for mapping orientation to colors. Boy's surface is the only real projective plane immersion without singularities. Coloring based on this model (we will refer it as *RP2 coloring*) will map different orientations to different and similar colors, except along the self intersection curve.

Methods:
We use Bryant-Kusner parameterization of Boy's surface (see Figure 1). For each orientation primitive (i.e., principal eigenvector) we evaluate the Bryant-Kusner parameterization of Boy's surface in the RGB color space. Fig 2 shows the top view of the half unit sphere where point colors obtained by evaluating them on Boy's surface. Fig 3 shows the half unit sphere with colors obtained by assigning the absolute coordinate values to RGB colors, where coloring is not one-to-one obviously (typical areas of such shown within circles).

Results & Discussion:
We applied both our coloring method (Fig4) and the conventional RGB coloring (Fig 5) on a coronal slice of a public DTI data set belonging to a monkey. In Figure 5, it is clear that the RGB mapping colors symmetric orientations with the same color, our coloring method distinguish them. Note that Boy's surface has a self intersection along a curve which intersects itself at a triple point. However, since we will get unique colors most of the time; it will be easy to distinguish the rare double points from the context.

References:
[1] S. Pajevic and C. Pierpaoli. *Color schemes to represent the orientation of anisotropic tissues from diffusion tensor data.*,MRM,1999.
[2] Werner Boy, *Über die Curvatura Integra und dieTopologie der Geschlossener Flächen*, 1901.

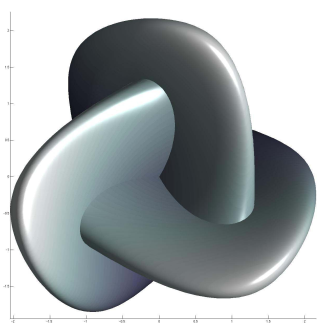

Fig1:Boy's Surface

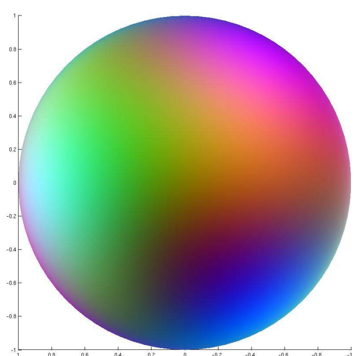

Fig2:RP2 coloring

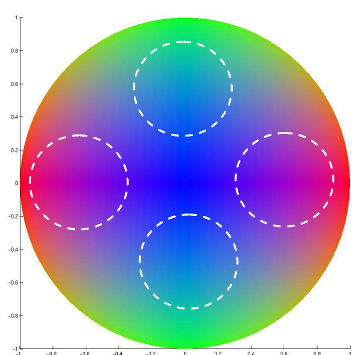

Fig3:RGB coloring

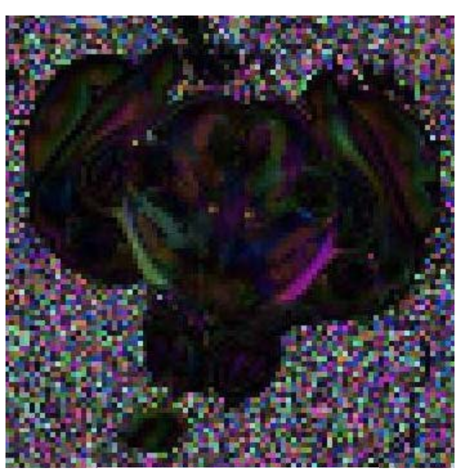

Fig4:RP2 coloring

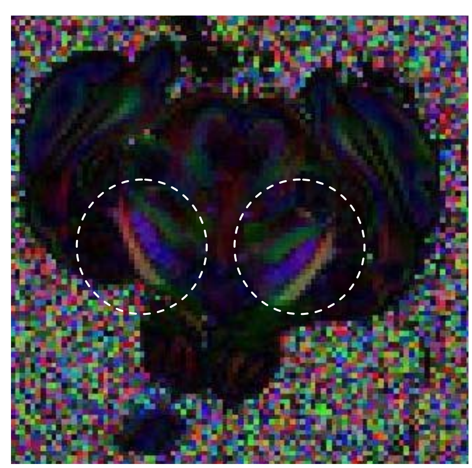

Fig5:RGB coloring