Optimization of undersampled variable density spiral trajectories based on incoherence of spatial aliasing

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Introduction: Variable density spiral (VDS) imaging typically samples low spatial frequencies densely and high spatial frequencies sparsely [1]. Reconstructed images are prone to aliasing artifacts, but they are considered to be acceptable in certain cases where aliasing of high spatial frequencies do not significantly deteriorate image quality. VDS imaging is particularly useful for dynamic MR applications such as cardiac imaging, where both high spatial and temporal resolution are desirable. However, unlike uniform density spiral (UDS) imaging, the VDS sampling density variation in k-space provides additional trajectory design parameters that have not, in the literature, been optimized in a systematic way. We identify the incoherency in spatial aliasing artifacts as an appropriate criterion for optimizing the trajectory design. Regularized iterative reconstruction is used to further reduce aliasing artifacts resulting from the "optimally" undersampled VDS trajectory [2]. We demonstrate the effectiveness of this approach in phantom studies.

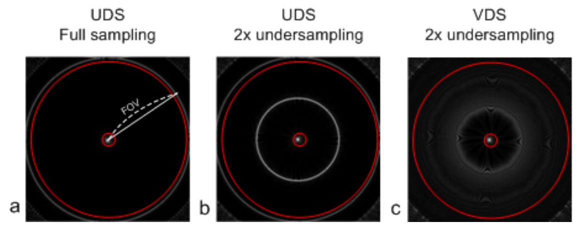

Figure 1: Point spread functions in uniform and variable density spiral imaging. The sidelobe region of interest (ROI) is the region between the inner and outer red circles. From the intensity distribution within the sidelobe ROI, aliasing energy is (a) fully suppressed, (b) coherent, and (c) incoherent.

Theory: VDS sampling density denoted by field-of-view (*FOV*) is parameterized as a function of k-space radius: $FOV(k_r;\ \{F_j\})$, where $0<k_r<k_{r\max}$, and $\{F_j\}$ is a set of parameters that determine *FOV*. Time-optimal k-space trajectories are designed using the density variation parameters $\{F_j\}$ and $k_{r\max}$. The PSF is computed using direct Fourier transformation:

$$PSF(r;\{F_j\}) = \sum_{n=1}^{N} DCF(k_n) e^{i2\pi k_n \cdot r}, \qquad (1)$$

N is the total number of k-space samples, $\{k_n\}_{n=1,\dots,N}$, is the set of normalized k-space locations, and $DCF(k_n)$ is the appropriate density compensation factor at k_n. Given N, the proposed optimization problem is written as:

$$\{\hat{F}_j\} = \arg\min \max_{r\in S} \left|PSF(r;\{F_j\})\right|, \qquad (2)$$

S is the sidelobe ROI (see Fig 1). Maximum intensity in the sidelobe ROI is used as an indicator of incoherency of spatial aliasing.

Methods: A polynomial function was used when designing the density variation: $FOV_\alpha(k_r) = \sum_{j=0}^{\alpha} F_j \left(\frac{k_r}{k_{r\max}}\right)^j$.

Monotonically decreasing linear (α=1) and quadratic (α=2) variations were considered. Given the spatial resolution, $1/k_{r\max}$, the number of k-space samples per interleaf, the number of interleaves, and gradient hardware limits, a set of VDS trajectories was obtained by using previously developed VDS design software [3]. Monte-Carlo simulation was performed to find the optimal VDS trajectory based on Eqn. (2). Voronoi density compensation factors [4] were used to compensate for non-uniformly spaced sampling density in the calculation of the PSF and in the gridding reconstruction. Regularized iterative reconstruction was performed by minimizing a cost function, which consists of data consistency and total variation penalty terms [2].

Experiments were conducted on a GE Signa EXCITE 3.0T system using a single channel transmit/receive head coil. Imaging parameters were: spiral gradient echo, 1820 samples per interleaf, 0.6 x 0.6 mm^2 spatial resolution, FOV = 12 cm / 24 interleaves for fully sampled UDS trajectories. Optimal VDS trajectories indicated by small circles in Fig. 2 were used.

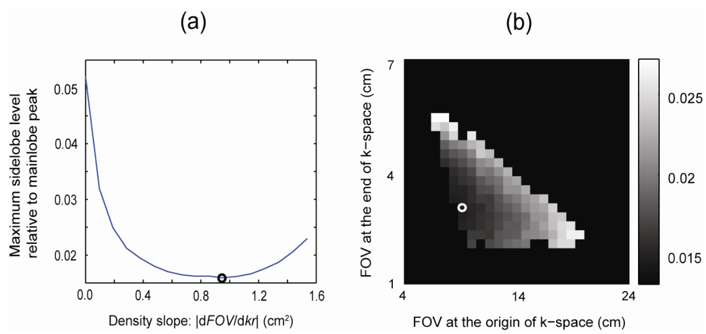

Figure 2: Cost function (peak sidelobe intensity) as a function of FOV variation parameters for (a) linearly and (b) quadratically decreasing sampling density.

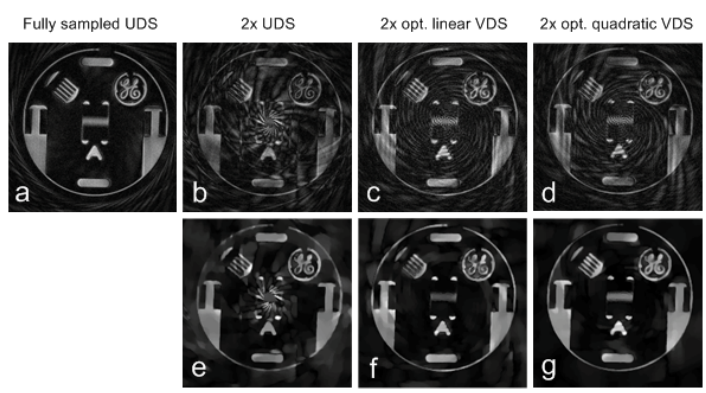

Figure 3: Axial images reconstructed from a resolution phantom. Gridding reconstruction (top row) and iterative reconstruction regularized with total variation penalty (bottom row).

Results and Discussion: The cost function plots in Fig. 2 show convex patterns and indicate that maximally dense sampling of low spatial frequencies in k-space does not lead to optimal suppression of the maximum PSF sidelobe intensity. The minimum values for the linear and quadratic variations were 0.0159 and 0.0148, respectively (see Fig. 2a,b). Reconstructed images have severe aliasing artifacts in the undersampled UDS trajectory (Fig. 3b,e). Our qualitative interpretation of the optimal VDS images is that gridding reconstruction produces images with reduced aliasing (Fig. 3c,d), and total variation regularized iterative reconstruction produces substantially reduced aliasing artifacts (Fig. 3f,g).

Conclusion: An optimization of sampling density variation in VDS imaging has been proposed. The approach promotes incoherency of spatial aliasing. Its effectiveness has been successfully verified in a resolution phantom. This optimized VDS design may be applicable to compressed sensing MRI, where the incoherency of aliasing due to undersampling is desired.

References: [1] Tsai et al., MRM 43:452-458, 2000, [2] Lustig et al., MRM 2007 (in press), [3] Hargreaves B, www-mrsrl.stanford.edu/~brian/vdspiral, [4] Rasche et al., IEEE TMI 18:385-392, 1999.