# Fast Simultaneous Measurement of the RF Flip Angle and the Longitudinal Relaxation Time for Quantitative MRI 

J-J. Hsu ${ }^{1}$, G. Zaharchuk ${ }^{\mathbf{1}}$, and G. H. Glover ${ }^{1}$<br>${ }^{1}$ Lucas Center for Imaging, Stanford University, Stanford, California, United States

Introduction. In quantitative MRI, measuring the flip angle $\alpha$ of an RF pulse and the longitudinal relaxation time $T_{1}$ is of great importance because, for example, the pixel intensity of images obtained by steady-state pulse-train methods is a function of both $\alpha$ and $T_{1}$ as given in the formula [1] leading to the calculation of the Ernst angle. Mapping the flip angle and $T_{1}$ are needed especially in longitudinal studies where the baseline reference of image intensity must be maintained constant throughout, and in imaging at high field, where the flip angle is instrumentally difficult to be made uniform across the imaged volume. Conventional methods for flip-angle mapping derive $\alpha$ using trigonometry identities from pixel intensity of two (or more) flip angles but delay for full magnetization recovery is required, making them time-consuming (e.g., $\sim 2 \times 5 T_{1} N$ for collecting $\alpha$ and $2 \alpha$ images, where $N$ is the number of repetitions). Even longer time is needed for the $T_{1}$ mapping. In this work, two novel methods are developed which can acquire multi-slice data in a shorter amount of time $\left(\sim 5 T_{1} N\right)$ and both $\alpha$ and $T_{1}$ can be determined. The method will be of great interest in pixel intensity normalization, pulse sequence parameter optimization, dynamic contrast-enhanced MRI, perfusion MRI, etc.
Methods. §Theory [2,3].-Consider a sequence of images, each reconstructed from the signal of the corresponding RF pulse in a pulse train of common flip angle $\alpha$. Then for two identical pulse trains, one of them starting with inverted magnetization, subtraction between their sequence of images results in a sequence of images whose pixel intensity $S_{i}$ satisfies $S_{i}=S_{0} \exp \left(-t_{i} / T_{1}\right) \cos ^{i-1} \alpha$ (referred to as the base equation), where $i$ is the index of the image in the sequence ( $i \geq 1$ ), and $t_{i}$ the time coordinate of the $i$-th image and $S_{0}$ the magnitude at the time-coordinate origin. §Method I: flip-angle priority ( $\boldsymbol{F A P}$ ).-Suppose two such subtracted image sequences, both having an identical set of time coordinates $\left\{t_{i}\right\}$, are obtained. One sequence has flip angle $\alpha$ and the other has $2 \alpha$. Divide the first sequence by the second one and let $\zeta_{i}$ be the resultant ratio for a given pixel of the $i$-th images. From the base equation, $\zeta_{i}$ satisfies $\zeta_{i}=\zeta_{0}(\cos \alpha / \cos 2 \alpha)^{i-1}$. Regression of $\ln \left(\zeta_{i}\right)$ versus $(i-1)$ will be a straight line whose slope $b$ is $\ln (\cos \alpha / \cos 2 \alpha)$. Then $\alpha$ can be derived from $b$ and with $\alpha$ now known, $T_{1}$ can be obtained by curve-fitting the base equation. §Method II: relaxation-time priority ( $\boldsymbol{R T P}$ ). -Parallel to FAP, suppose two subtracted image sequences are obtained, both having a common flip angle $\alpha$. One sequence has time coordinate set $\left\{t_{i}\right\}$ and the other has $\left\{t_{i}^{\prime}\right\}$. Let $\xi_{i}$ be the ratio of the $i$-th images. From the base equation, $\xi_{i}$ satisfies $\xi_{i}=\xi_{0} \exp \left[\left(t_{i}^{\prime}-t_{i}\right) / T_{1}\right]$. Regression of $\ln \left(\xi_{i}\right)$ versus $\left(t_{i}^{\prime}-t_{i}\right)$ will be a straight line whose slope is $1 / T_{1}$. With this $T_{1}$ value, $\alpha$ can be obtained by curve-fitting either of the subtracted image sequences to the base equation. §Experimental.-A spiral scan [4] was used to collect the $k$-space data ( $128 \times 128$-matrix equivalent). A pulse train $\pi$ - $\alpha_{1}-$ $\alpha_{2}-\alpha_{3}-\alpha_{4}-\alpha_{5}-\alpha_{6}$ with $t_{i} \in\{0,500,1000,1500,2000,2500\} \mathrm{ms}$ was executed and $\alpha_{1}-\alpha_{2}-\alpha_{3}$ and $\alpha_{4}-\alpha_{5}-$ $\alpha_{6}$ were used as two identical trains; subtraction between their images generated the desired images at $t_{i} \in\{0,500,1000\} \mathrm{ms}$ that satisfy the base equation. For FAP, the above pulse train was repeated with the flip angle doubled. For RTP, a second pulse train with $t_{i}^{\prime} \in\{0,1100,1700,2200,3300$, $3900\} \mathrm{ms}$ was executed to obtain the desired subtracted images at $t_{i}^{\prime} \in\{0,1100,1700\} \mathrm{ms}$. Flip-angle mapping by using a conventional method [5] described in the Introduction was also performed. Three $5-\mathrm{mm}$ slices were acquired (explained in Ref. [3]). A phase-encoding gradient in the slice direction was included; each scan was repeated to Fourier-encode eight sub-slices covering a range double the slice thickness. For each slice, the central sub-slice was used in the result analysis to avoid the effect of imperfect slice profile. A spherical phantom made from solidified, uniform agar gel, was used for benchmarking. Volunteer scans were conducted in accordance with a protocol approved by the Stanford IRB.

Results and Discussion. Fig 1 shows flip-angle maps of the phantom at $1.5 \mathrm{~T}\left(\alpha=35^{\circ}\right.$ was specified; only one slice is shown). Two dimensional moving averaging over a box of $11 \times 11$ pixels is applied for smoothing. After smoothing, the average (standard deviation) from the conventional method is $31.8^{\circ}\left(1.0^{\circ}\right)$. By subtracting the smoothed map of FAP by that of the conventional method, the average pixel-by-pixel difference is $+1.1^{\circ}\left(0.4^{\circ}\right)$. The corresponding difference for RTP is $+2.3^{\circ}$ $\left(1.2^{\circ}\right)$. Repeated experiments with two volunteers have been performed successfully. Fig 2 shows a sample result for the brain obtained at $3 \mathrm{~T}\left(\alpha=35^{\circ}\right.$ specified). In the boxed area as shown in Fig 2(d), the average flip angle is $34.8^{\circ}\left(0.6^{\circ}\right)$ for the conventional method, $35.1^{\circ}\left(0.5^{\circ}\right)$ for FAP, and $36.8^{\circ}$ $\left(0.8^{\circ}\right)$ for RTP; the average $T_{1}$ is $851(41) \mathrm{ms}$ for FAP and $892(38) \mathrm{ms}$ for RTP, in agreement with the literature values of the white matter (e.g., Ref. [6]). In summary, both our methods are consistent and their flip-angle results are in agreement with the conventional method. Effects of signal-to-noise ratio and the linearity between $\alpha$ and $2 \alpha$ pulses are currently under study. Additional features can be implemented in our methods without extra scan time. For example, if the echo time of one of the pulse trains is delayed, the $B_{0}$ map can be derived from the delay time and the difference of the phase images. In certain array-coil applications, the images can also be used to derive the sensitivity maps.

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