Azimuthally Symmetric IBEM Gradient and Shim Coil Design

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Introduction Boundary element methods are well suited to solution of electromagnetic problems in the low frequency regime that is relevant for gradient and shim coil design. In previous work, inverse boundary element methods (IBEM) [1,2] have been shown to allow the design a variety of high performance gradient and shim coils on arbitrarily shaped surfaces, including domes and slotted cylinders [3]. In this approach, the coil surface is meshed into triangular elements and the coil characteristics are specified in terms of the stream-function values at nodes located at the vertices of the triangular elements. Optimal stream-function values are set by minimising a functional formed from the deviation of the field from the desired form over a set of target points and a combination of other coil characteristics including power dissipation, torque and stored energy. While this approach is highly versatile, the large number of mesh elements needed to define the coil surface means that implementation of IBEM imposes heavy demands on computational effort and available memory. Many coil systems required for MRI have a high degree of symmetry with the coils often being wound on a "surface of revolution" about the main field direction and the desired field variation generally has a known simple azimuthal dependence of the form cos($m\varphi$) or sin($m\varphi$). Incorporation of this *a priori* knowledge of the symmetry into the IBEM framework allows the coil to be described by a one-dimensional mesh, thus greatly reducing the number of elements and consequently the computational effort and memory requirements [4]. We have implemented this azimuthally symmetric IBEM for coil design and demonstrate its efficacy by designing two complex gradient and shim coil sets.

Methods In the azimuthally symmetric IBEM, the line whose rotation about the z-axis forms the coil surface is represented by the connection of a series of N nodal points. The stream-function values at these points, Ψ_n, define the current density over the whole coil surface in terms of a series of basis functions [1,2,3]. Figure 1 shows the two boundary elements that are associated with the n^{th} node as well as the cylindrical (ρ, φ, z) and elemental (λ, φ) coordinate systems. The magnetic field, B_z, at set of Q points (ρ_q, 0, z_q) in the φ=0 plane and the power dissipation, P, due to each basis function were calculated using Matlab®. The resulting system of equations was inverted to yield the optimal Ψ_n and thereby the wire paths of the coil design. The inductances and resistances of the coils were estimated using FastHenry©.

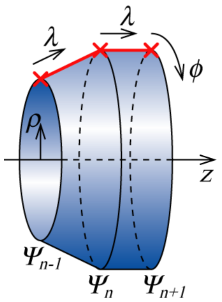

Figure 1. Geometry and coordinate systems at the n^{th} node.

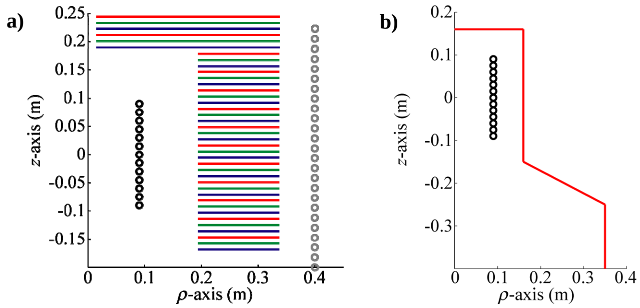

Figure 2. Geometry of the a) stacked annuli X- (red), Y- (green) and Z-gradient (blue) coils and b) bi-radial shim coil (red) with their target field point shown in the region of uniformity (black) and shielding (grey).

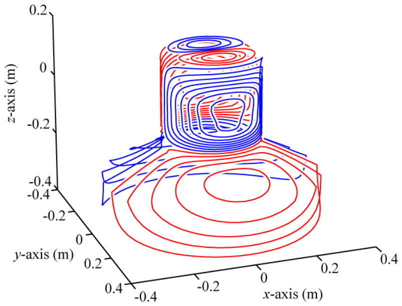

Figure 3. Wire-paths of the bi-radial ZX-shim coil (red wires have reversed current with respect to blue).

Two different coil geometries were used to demonstrate the efficacy of the azimuthally symmetric IBEM: (i) a 3-axis, insert head gradient coil set in which each coil was formed from 13 stacked annuli (comprising 253 nodes), whose dimensions were based on a previously described short coil set employing co-axial return paths [5]; (ii) a bi-radial shim coil (with a surface comprising 63 nodes). Figures 2 a) and b) show the two different coil geometries. The bi-radial shim coils were also modelled using the triangular element IBEM [1,2,3] for comparison (comprising 4553 nodes for comparable mesh detail).

Results Figure 3 shows the wire paths of a bi-radial ZX-shim coil and Fig. 4 shows the wire paths for each layer of the X-gradient coil composed of stacked annuli. Table 1 presents the performance data for these coils. The stacked annuli X, Y and Z gradient coils had η^2/L figures-of-merit (FOM) that were 3.48%, 375% and 3.6% greater than the equivalent coils in Ref. [5]. Actively shielded versions of these coils had FOMs of 3.4×10^{-5}, 3.9×10^{-5} and 6.3×10^{-4} $\text{T}^2\text{m}^{-2}\text{A}^{-2}\text{H}^{-1}$. The B_z and P matrices used in the IBEM for the bi-radial coil were calculated in 153 s and 0.2 s respectively. To give similar mesh detail it was necessary to use 8640 triangular elements [1,2,3] and the equivalent calculations took 4639 s and 6882 s. The memory required by the matrix of the system is 0.08 and 152 MB for the azimuthally symmetric and triangular element IBEM approaches.

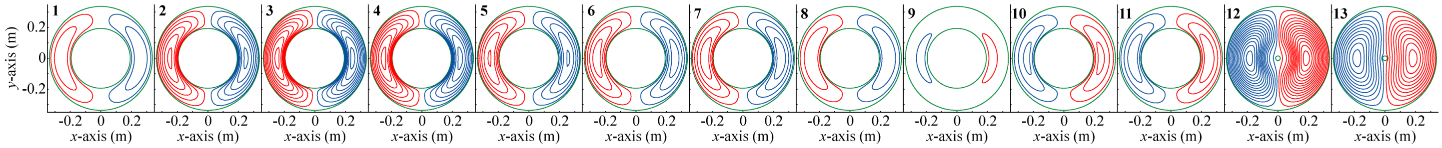

Figure 4. Wire-paths of the 13 layers of the stacked annuli X-gradient coil (red wires have reversed current with respect to blue).

Coil	Stacked Annuli X	Stacked Annuli Y	Stacked Annuli Z	Biradial ZX Shim
N	14	14	10	8
η ($\text{mTm}^{-n}\text{A}^{-1}$)	0.23	0.23	0.59	1.16
ΔB_z (%)	4.8	4.7	4.1	5.0
Δw_{min} (mm)	3.1	3.4	8.5	6.6
L (μH)	615	565	677	77
R (mΩ)	347	325	239	74
η^2/L ($\text{T}^2\text{m}^{-2n}\text{A}^{-2}\text{H}^{-1}$)	8.7×10^{-5}	9.2×10^{-5}	5.2×10^{-4}	1.7×10^{-2}

Table 1. Properties of the coils used as examples of the axially symmetric IBEM method, including the number of stream-function contours, N, efficiency, η, field error, ΔB_z, minimum wire spacing, Δw_{min}, inductance, L, resistance, R, and FOM, η^2/L.

Conclusions Incorporation of *a priori* information about coil symmetry allows a significant optimisation of the IBEM approach, speeding up the coil design process and reducing computational demand. Use of this azimuthally symmetric IBEM has allowed the design of novel head insert gradient coils that are composed of stacked annuli. These fingerprint-type designs have significantly better performance than coils made of simple arc and radial wire units [5]. It was found that designing these coils using triangular elements [1,2,3] was not feasible due to the large numbers of nodes (> 5000) needed to give acceptable mesh detail. Successful design of a ZX shim coil of bi-radial geometry demonstrates that the modified IBEM approach can be used to design coils of non-standard geometry which generate complex field patterns. Incorporation of stored energy minimisation into this framework is planned, which will further improve the coils' FOM.

References **[1]** S. Pissanetzky, *Meas. Sci. Technol.*, **3**, 667-673 (1992). **[2]** R. Lemdiasov and R. Ludwig, *Concepts MR: B*, **26**, 67-80 (2005). **[3]** M. Poole and R. Bowtell, *Concepts MR: B*, **31**, 162-175, (2007). **[4]** G. Peeren, *J. Comput. Phys.*, **191**, 305-321, (2003). **[5]** R. Bowtell and A. Peters, *Magn. Reson. Med.*, **41**, 600-608 (1999).