The Equivalent Magnetizing Current (EMC) method for biplanar active and passive shim design

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Synopsis: This paper presents a new method for biplanar active and passive shim design using an Equivalent Magnetizing Current (EMC) method. The EMC induced by the rotational component of the magnetization is equivalent to that of the stream function (SF) and hence the SF is proportional to the magnetization. Using this approach, the magnetic field generated by a magnetized disk of finite thickness is related directly to the SF and hence no intermediate step to transform the current density into SF is required. Optionally, instead of a current pattern, a set of iron pieces can be employed so that the magnetized shims can be placed at equally spaced contours of the magnetization-stream function (MSF). The MSF is expressed as a sum of orthogonal functions of the azimuthal angle and shim domain radius and so it is tailored in the source domain in order to generate a particular magnetic field harmonic or a combination of these inside the DSV. The method is validated using known examples and the potential to generate new solutions is demonstrated.

Method: We assume an isotropic, non-hysteresis and homogenous ferromagnetic disk of radius ρ_{max}, uniformly magnetized ($M_z\mathbf{e}_z$) along the axial axis z. See Fig. 1. The disk thickness t is much smaller than the axial distance ($Z_0^{\pm}$) between the two pole faces and thick enough to produce an induced magnetization M_z along the axial axis. The signs (+ and -) identify the upper and lower pole faces. We consider the disk immersed in a homogeneous magnetizing field $H_c\mathbf{e}_z$ and M_z depends on the position (ρ',ϕ') in the disk domain. From the assumption we have $\mathbf{M}\times\mathbf{e}_z=0$ and hence vector potential produced by the magnetized disk can be written as:

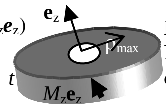

Fig.1- Magnetized disk (+).

$$\mathbf{A}=\frac{\mu_0}{4\pi}\int_{V'}\frac{\nabla'\times\mathbf{M}}{R}\cdot t\,ds'$$ where $\mathbf{J}_v=\frac{1}{\mu_0}\nabla\times\mathbf{M}$ is the volume current density equivalent to the magnetization induced in the iron disk [1]. $\mathbf{J}_v(\rho',\phi')$ and $\mathbf{M}(\rho',\phi')$ do not depend on z' and thus $\nabla\cdot\mathbf{M}=0$, which indicates that the magnetization is a solenoidal field that flows in a very thin disk of volume $dV'=\rho' t\cdot ds'$. Then we can assume that $^{*}\mathbf{J}_m=\frac{t}{\mu_0}\nabla\times\mathbf{M}$ which is given in [A/m]. Due to the continuity equation ($\nabla\cdot\mathbf{J}_m=0$), the current can be expressed as the curl of the vector **S** ($^{**}\mathbf{J}_m=\nabla\times\mathbf{S}$) where in a planar surface $S\mathbf{e}_z$ is the effective component of the vector **S** that produces the current $\mathbf{J}_m$. Comparing equations (**) with (*) we obtain $\mathbf{S}=t/\mu_0\mathbf{M}$. The curl of the magnetization (**M** or vector **S**) generates an equivalent current with a vector potential that has a single component S pointing in the outward axial direction. Thus, we can write the SF as:

$$S(\rho',\phi')\mathbf{e}_z=\frac{t}{\mu_0}M_z(\rho',\phi')\mathbf{e}_z$$; which is given in [A] units.

Expressing the magnetic field generated by the magnetized element in terms of spherical harmonics and separating the source information from the field spatial dependence we can write the coefficients corresponding to the magnetic field oscillating harmonics as:

$$A_{nm}=\int_{\rho_{min}}^{\rho_{max}}\int_0^{2\pi}C_{nm}(\rho',\phi')\cos m\phi'\,d\rho'd\phi' \text{ and } B_{nm}=\int_{\rho_{min}}^{\rho_{max}}\int_0^{2\pi}C_{nm}(\rho',\phi')\sin m\phi'\,d\rho'd\phi',$$ where C_{nm} contains the MSF $S(\rho',\phi')$. The magnetization is defined as: $$M_z(\rho',\phi')^{\pm}=f(\rho',\rho_{max})\sum_{n'=1}^{N}\sum_{m'=0}^{M}h(a_{m'}^{\pm},b_{m'}^{\pm},c_{n'}^{\pm}d_{n'}^{\pm},\rho',\phi').$$

The parameter h is expressed as an oscillating component multiplied by the unknown amplitudes ($a^{\pm}_{m'}$,$b^{\pm}_{m'}$,$c^{\pm}_{n'}$,$d^{\pm}_{n'}$). The parameters N and M define the number of modes along the radial and azimuthal direction, respectively. The MSF must be a differentiable and continuous function within the disk domain and must be defined in such a way that $\nabla\cdot\mathbf{J}_m=0$. For a ferromagnetic body of volume V limited by the surface S', where no volumetric charges exist ($\nabla\cdot\mathbf{M}=0$), the magnetostatic energy W_m can be written as [1]:

$$W_m=\frac{1}{2}\int_{S'}\int_{S''}\frac{M_z(\rho',\phi')M_z(\rho'',\phi'')ds'ds''}{|r'-r''|}.$$

Instead of expressing the magnetic field as function of one of the **J** components, the magnetic field is written in terms of the MSF and the energy is expressed in terms of the magnetization. This is one of the main distinguishing features of the proposed method over previous approaches [2]. Another interesting attribute of the present method is that the magnetic field kernel to calculate the contribution of each mode for a given harmonic is the magnetic field generated by a magnetized iron volume.

The active/passive shimming design method searches for amplitudes ($a^{\pm}_{m'}$,$b^{\pm}_{m'}$,$c^{\pm}_{n'}$,$d^{\pm}_{n'}$) that minimize W_m and at the same time produces the target harmonic within the tolerance ε in the DSV. The optimization problem can be written as follow:

$$\min W_m$$
subject to
$$-\varepsilon\le A_{nm}+A_{nm}^{\text{target}}\le\varepsilon$$
$$-\varepsilon\le B_{nm}+B_{nm}^{\text{target}}\le\varepsilon$$
$$-S_{max}\le S(\rho',\phi')^{\pm}\le S_{max}$$

where (A_{nm}^{target} and B_{nm}^{target}) are the target harmonics and S_{max} is the maximal permissible value of the thickness-magnetization proportion or the maximal permissible value of the stream function in all the disk domain. After determining the amplitudes ($a^{\pm}_{m'}$,$b^{\pm}_{m'}$,$c^{\pm}_{n'}$,$d^{\pm}_{n'}$) the current pattern is generated using the SF S [2]. If iron is used to generate the target harmonic then magnetized shim pieces are placed at equally spaced contours of S using the same thickness t. If it is not possible to reverse the magnetization direction then iron pieces are distributed in the positive domain of the MSF map. Using the MSF map as initial solution, an LP algorithm can be used to find the location and the minimal thickness-magnetization value for a discrete shim array to generate a target harmonic. The method is geometry independent and can be extended to 3D cases.

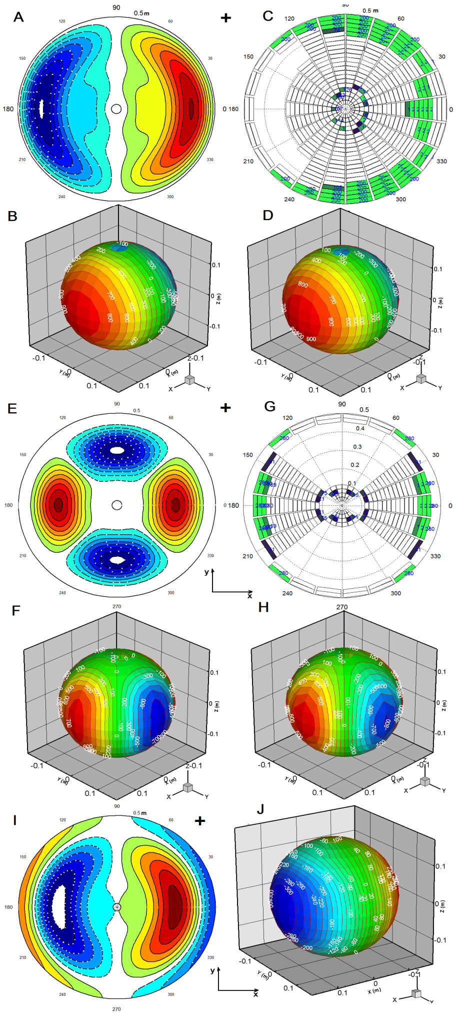

Fig 2. (A) One half (+) of a conventional active shim profile to produce a gradient along the x axis. (C) The equivalent passive shim design to generate a x-axis gradient. (E,G) One half of an active and the equivalent passive shim profiles to generate A_{22}. (B,D,F,H) The magnetic field profile given is in μT. (I) An unconventional unshielded gradient x coil and the magnetic field profile at the DSV (J). Dashed line represents the reversed current.

Results and Discussions: To illustrate the flexibility of the method in generating a target harmonic, two conventional active shims and the equivalent passive shim profiles that produce A_{11}=-1005 μT and A_{22}=-275 μT have been designed. The DSV was 15.5 cm, $Z_0^{\pm}$=46 cm, ρ_{max}=50 cm, ρ_{min}=2.5 cm. The operating current was I=200 A for the active shim and for the equivalent discrete passive shim we used saturated (M_z=2.144 T) soft iron with no reversible magnetization direction. Fig. 2-(A,C,E,G) shows the results. The coil (A) produced A_{11}=-1011.14 μT; the largest remaining harmonic was A_{51}=3.027 μT. The profile (G) produced A_{11}=-1003.56 μT using 11 kg of iron per pole; and the largest remaining harmonic was A_{40}=1.22 μT. Fig. 2-(B,D) shows a linear magnetic field along x axis produced by the active and the passive shim. The coil (E) produced A_{11}=-275.43 μT with a remaining A_{42}=0.63 μT. The iron array (G) produced A_{11}=-275.1 μT with a remaining A_{20}=-0.87 μT using 6 kg of iron per pole face. Fig. 2-I, presents a novel x-axis gradient coil that produces 0.02 mT/m·A. The returning current path can be arranged to flow towards the shielding coil. In all the cases the remaining (impurity) harmonics were more than 330 times smaller than the target harmonic.

Conclusions: A new alternative method for biplanar active and the equivalent passive shim design has been presented using a common formulation. Applying the EMC concept, a MSF map is generated where current or iron patterns can be placed at equally spaced contours of the MSF. The method can be a versatile tool for designing hybrid MRI systems (current/iron) using the MSF map to define the shape, strength and location of the field sources (iron and/or current). Target harmonics with very small impurities are produced by the method. Novel active/passive shim solutions can be obtained using the EMC method.

References: [1] J. D. Jackson. John Wiley & Son, 1998. [2] M. A. Brideson, et al. *Concepts Magn Reson*, vol. 14, pp. 9 – 18, 2002.

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