MRI Compressed Sensing via Sparsifying Images

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Introduction: Recently, there has been an emerging interest to accelerate MRI through iterative reconstruction of undersampled data based on metrics that promote image sparsity. Compressed sensing theory predicts that such methods may accurately reconstruct images from data sampled much below the Nyquist limit through minimization of L1 norms (sum of the absolute values of all pixels) [1,2]. The other conditions require that the *k*-space trajectory should provide non-coherent aliasing patterns and that the underlying image must be sparse. It has been shown that the radial trajectory satisfies the first condition well [3]. If the image is not sparse, a sparsifying transformation should be used to allow compressed sensing. An example of such sparsification is the gradient transformation used in total variation (TV) reconstruction [4]. The challenges of the compressed sensing include applicability to complex-valued MR images and the smoothing effect of the gradient operation.

In this work, we extend the compressed sensing framework via *sparsifying images*. We demonstrate that such an approach may be a powerful tool for image reconstruction from highly undersampled data. The method utilizes the recent idea in HYPR methods [5] to use sliding window composite images to constrain reconstruction. At the same time, such enhancement is done within the compressed sensing framework. Hence, we have called the method HighlY constrained back PRojection by Iterative esTimation (HYPRIT). As an example, we demonstrate the potential of the method for accelerated radial diffusion tensor imaging (DTI).

Theory: To estimate the underlying image $\mathbf{f}$ from an incomplete dataset along compressed sensing guidelines, the following mathematical problem should be solved:

$$\arg\min_{\mathbf{x}}\left(\|\mathbf{Ef}-\mathbf{s}\|_2+\lambda\|\mathbf{\Psi f}\|_1\right). \quad [1]$$

Here, $\mathbf{E}$ is the encoding matrix, $\mathbf{s}$ is data vector, and $\mathbf{\Psi}$ is sparsifying transform matrix, such as image gradient $(\mathbf{\Psi}=\mathbf{D})$, $\mathbf{D}$ is matrix of first order image differences, or discrete wavelet transform, λ is the regularization parameter. Often, data are sampled in a temporal or parametric dimension and possess a significant degree of redundancy, as image pixels may highly correlate along such a dimension. For example, in time-resolved contrast enhance MRA, a large number of pixels are stationary. In DTI, gray matter pixels for different diffusion gradient directions are highly correlated because of highly isotropic diffusion in gray matter. We propose to use this property to enforce sparsity for compressed sensing reconstruction as follows:

$$\arg\min_{\mathbf{x}}\left(\|\mathbf{Ef}-\mathbf{s}\|_2+\lambda\|\mathbf{\Psi}(\mathbf{f}-\mathbf{f}_S)\|_1\right). \quad [2]$$

Here, $\mathbf{f}_S$ is a sparsifying image which is a reasonable estimate of image intensity distribution. One possible choice is a composite image $\mathbf{f}_C$ obtained by sliding window reconstruction of an interleaved radial dataset. The whole procedure is illustrated in Fig. 1.

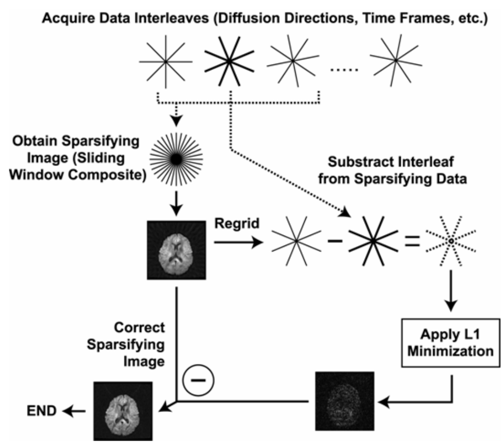

Figure 1. Diagram of HYPRIT method.

Methods and Results: Evaluation of HYPRIT was done on diffusion weighted images (12 different diffusion encoding directions). An iteratively reweighed approach was implemented for the L1 norm minimization stage of the procedure [6]. Figure 2 compares the sparsifying properties of different methods for DTI data. The results show that images after HYPRIT sparsification $(\mathbf{\Psi}=\mathbf{I},\mathbf{f}_S=\mathbf{f}_C)$ and TV-HYPRIT sparsification $(\mathbf{\Psi}=\mathbf{D},\mathbf{f}_S=\mathbf{f}_C)$ are sparser than with the TV approach alone $(\mathbf{\Psi}=\mathbf{D},\mathbf{f}_S=0)$, both visually and in terms of the L1 norm, the measure of image sparsity. To illustrate the potential of the proposed approach, HYPRIT was applied to the radial DTI data simulated from the DW images. The undersampled radials (speedup factor 20) were interleaved for different diffusion gradient directions. We applied a temporal filter to obtain the composite images from the interleaved radial data [7]. The reconstruction produced high SNR diffusion tensor color maps (Fig. 3a) and preserved major WM tracts as compared to the reference image in Fig. 3c. For comparison, gridding reconstruction of undersampled data is shown in Fig. 3b.

Discussion: Compressed sensing is a powerful framework for MR image reconstruction from highly undersampled data. In this work, we have proposed an extension of the framework, which makes use of data sampled in the time/parametric direction for efficient compressed sensing. This compressed sensing approach does not rely on specific assumptions of other techniques such as the piecewise constant model of MR images and image phase smoothness (TV approach, [4]). At the same time, it learns about information redundancy in the multiple datasets and utilizes this information to implement compressed sensing concepts.

We have demonstrated the HYPRIT algorithm with radial trajectories, which are advantageous for two reasons. First, it provides incoherent aliasing artifacts. Second, it fully samples the k-space center and thereby allows applying compressed sensing only to the unsampled higher spatial frequencies (Fig. 2b). We expect the results to hold for other interleaved non-Cartesian trajectories such as spiral trajectory, with more acceleration available with their self-calibration versions.

Currently, the method is being extended for 3D DW data [8]. The proposed approach may be useful for a number of methods including different quantitative MRI techniques. The particular applications of the technique should be justified by physical properties of acquisition. For example, the success of the method for DTI data reduction is ensured by the fact that diffusion in GM is highly isotropic and hence does not differ significantly for different gradient encoding directions. In the case of T2 relaxation fits to multi echo CPMG data, tissues with longer T2 such as CSF will experience fewer changes from echo time to echo time and will be also highly correlated. Alternatively, the signal decay model could be incorporated into the estimate to allow direct parameters evaluation of T2 decay parameters.

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References: [1] Donaho D, IEEE TIT 2006. 52:1289. [2] Candes E, et al. IEEE TIT 2006. 52:489. [3] Lustig M. et al. MRM In press. [4] Block KT, et al. MRM 2007. 57(6):1086. [5] Mistretta CA, et al. MRM 2006, 55:30. [6] Gorodnitsky *et al.* IEEE TSP 1997, 45(3): 600. [7] Liu et al. IEEE-TMI 2006. [8] Jung YK, et al. ISMRM 2007.

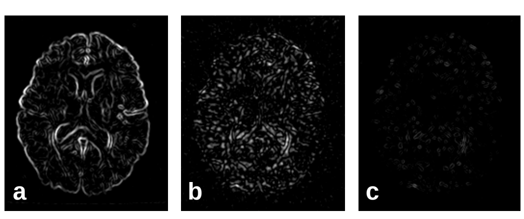

Figure 2. DW image after sparse transformations. **a**: Image gradient (TV, L1 norm=6.7), **b**: sparsification by composite (HYPRIT, L1 norm=5.9). **c:** gradient on *b* (TV-HYPRIT, L1 norm=3.3). Decreasing L1 norm indicates that proposed sparsification may be stronger than with image gradients.

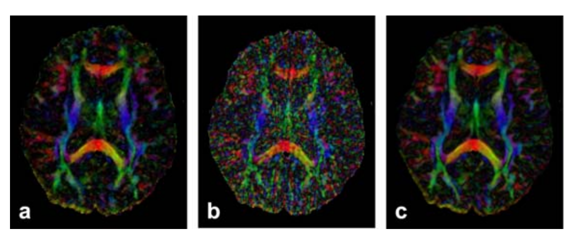

Figure 3. Application of HYPRIT to DTI data (simulated radial DTI data, undersampling factor 20, 12 diffusion directions). Diffusion color maps are shown for **a:** HYPRIT reconstruction, **b:** Gridding, **c:** Reference data.