Dyadic Green's functions for electrodynamic calculations of ideal current patterns for optimal SNR and SAR

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Introduction

Accurate modeling of electromagnetic (EM) effects is becoming increasingly important as higher magnetic field strengths are employed in MR systems. The interactions of the EM field with biological tissues at high frequencies require appropriate coil designs to improve image quality and to avoid adverse effects in patients. Modeling of the signal-to-noise ratio (SNR) has become therefore a common phase during the design of radiofrequency (RF) detector coils. On the other hand, evaluation of the specific absorption rate (SAR) is fundamental to assess potential health effects and compliance with safety standards. Rigorous and time-consuming numerical simulations with techniques such as the finite difference time domain (FDTD) technique are normally used for EM analyses with detailed heterogeneous models of the human head [1]. In this work we use mode expansions with dyadic Green's functions (DGF) [2] to express the full-wave EM field in a dielectric sphere. A similar DGF approach for SNR calculation was described by Schnell et al. in the case of a cylindrical sample [3], but to our knowledge such an approach has not been explored for spherical geometries until now. Semi-analytical calculations of SNR and SAR for simulated MR experiments, both for specific coil geometries and for the ultimate intrinsic case, can be performed quickly with our DGF formulation. The theoretical framework also enables derivation of optimized surface current patterns and includes as a special case a previously described theory of ultimate SNR [4].

Theory and Methods

Dyadic Green's functions associated with a dielectric sphere were defined as double Fourier series of vector wave functions in spherical coordinates as in Ref. [2]. Among the possible solutions, we chose $M_{l,m}(\mathbf{r},k) = j_l(k\rho)\mathbf{X}_{l,m}(\theta,\varphi)$ and $N_{l,m}(\mathbf{r},k) = (1/k)\nabla\times j_l(k\rho)\mathbf{X}_{l,m}(\theta,\varphi)$, where l, m are the expansion indices, k is the complex wave number, j_l is a spherical Bessel function of order l and $\mathbf{X}_{l,m}$ is a vector spherical harmonic. The Fourier series were weighted by appropriate coefficients to account for boundary conditions at the surface of the sphere. The DGF formalism enables calculation of the electric field resulting from any spatial current distribution **J** as: $\mathbf{E}(\mathbf{r}) = i\omega\mu_o\int_V \mathbf{G}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r})d\mathbf{r}'$, where ω is the angular frequency, μ_o is the magnetic permeability in free-space and $\mathbf{G}(\mathbf{r},\mathbf{r}')$ is the branch of the DGF associated with the region indicated by **r**. If we define the current distribution to exist only on the sphere surface, this expression reduces to a surface integral. In the most general case, the surface current density may consist of both magnetic-type and electric-type components, indicated with the superscript $^{(M)}$ and $^{(E)}$ respectively, and we can express it with a mode expansion. The generic surface current mode would take the form of $k_{lm} = W_{lm}^{(M)}\mathbf{X}_{l,m}(\theta,\varphi) + W_{lm}^{(E)}\hat{\rho}\times\mathbf{X}_{l,m}(\theta,\varphi)$, where $W_{lm}^{(M)}$ and $W_{lm}^{(E)}$ are the series expansion coefficients representing divergence-free and curl-free surface current contributions, respectively. Once the electric field is computed, the magnetic field can be derived as $\mathbf{B}(\mathbf{r}) = (-1/i\omega)\nabla\times\mathbf{E}(\mathbf{r})$. The complete set of current modes can be employed to calculate the ultimate intrinsic SNR, independent of any coil design, or these modes can be weighted appropriately and combined to derive the SNR of specific coil configurations. For Cartesian SENSE reconstructions [5], the SNR at a generic voxel n can be expressed as $SNR_n \propto \omega_o B_o \Big/ \sqrt{4k_B T_S (\mathbf{X}\mathbf{R}^{-1}\mathbf{X}^H)^{-1}_{nn}}$, where ω_o is the Larmor frequency, B_o is the main magnetic field strength, k_B is Boltzmann's constant, T_s is the temperature of the sample and the superscript ^{H} indicates a conjugate transpose. The matrix **X=TS** is the sensitivity matrix **S**, computed using the left circularly polarized component of the modes' magnetic field, multiplied by a transformation matrix **T** that accounts for boundary conditions at the surface of the sphere. The noise resistance **R** is calculated as $\mathbf{R} = \mathbf{T}\mathbf{R}_L\mathbf{T}^H + \mathbf{R}_A$, where $R_{L,(i,j)} = \int_V \sigma(\mathbf{r})e_j^*(\mathbf{r},t)\cdot e_i(\mathbf{r},t)d\mathbf{r}$ (with σ being the sample conductivity and $e_i(\mathbf{r},t)$ the electric field generated by a unit current on the ith mode) accounts for sample losses and $\mathbf{R}_A = 1/(d_C\sigma_C)\int_A k_j^*(\mathbf{r},t)\cdot k_i(\mathbf{r},t)\rho dA$ (with σ_C being the conductivity of the coil material, d_C its thickness and $k_i(\mathbf{r},t)$ the current distribution of the ith mode) accounts for losses in the conductors. If we remove $\mathbf{R}_A$ and allow a fully general set of current modes, the boundary condition matrix T disappears from the SNR denominator and the expression for ultimate SNR becomes identical to that derived by Wiesinger et al. using a multipole EM field expansion [4]. However, the DGF approach begins by defining current distributions, so it has the advantage that we can perform a weighted sum of the individual current modes k_{lm} using weights derived from the SENSE reconstruction and find the ideal surface current pattern that results in the ultimate SNR. The DGF formulation also extends naturally to SAR analysis in the transmit case, as the electric field resulting from an arbitrary current distribution can be applied directly to calculate RF power deposition in the object. In parallel transmission it is possible to combine the excitations of the individual transmit elements in a way that minimizes SAR [6] and in the case of rectilinear EPI-type excitation trajectories the optimal weights at any time point are computed with an inverse Fourier transformation: $F^{-1}\left\{\mathbf{R}^{-1}\mathbf{X}_n^H(\mathbf{X}_n\mathbf{R}^{-1}\mathbf{X}_n^H)^{-1}\mu_n\right\}$, where μ_n is the target profile at location n associated with the particular time point and **R** comprises only sample losses. The resulting optimal average global SAR over the entire duration of the pulse takes the form: $\xi = 1/N\sum_1^N\left[\mu_n^H(\mathbf{X}_n\mathbf{R}^{-1}\mathbf{X}_n^H)^{-1}\mu_n\right]$, where N is the total number of image voxels. If we use the complete basis of current modes, in place of actual coil current densities, we can calculate the ultimate intrinsic SAR [7] of the excitation. In the transmit case, ideal surface current patterns are calculated as a function of time, while traversing excitation k-space.

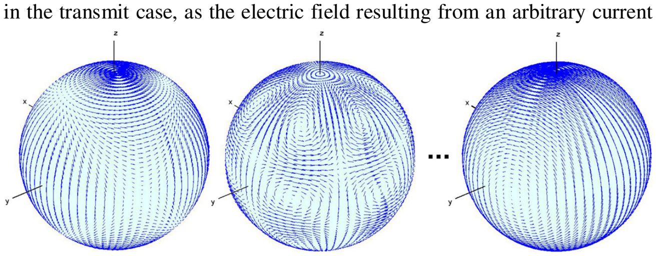

Fig. 1 Illustration of the process by which ideal surface current patterns are generated for the case of Bo = 3T. The first two plots from the left show optimal current patterns for two arbitrarily chosen modes (l=2,m=1 and l=5,m=3). The rightmost plot show the ideal current pattern, evaluated at every arrow position as the sum of the contributions of all current modes.

Results and Discussion

Figure 1 provides an example of the surface current patterns that can be calculated with the DGF formalism. The rightmost plot shows the net ideal surface current pattern resulting in ultimate intrinsic SNR at a voxel at the center of the sphere for 3T magnetic field strength. The ideal current patterns are derived by summing the contributions of the individual current modes, each weighted by the corresponding coefficient from the SNR-optimal reconstruction. For the leftmost plot and the central plot two current modes (order l=2,m=1 and l=5, m=3) were arbitrarily chosen as examples. The current patterns of the individual modes actually have more apparent structure than does the ideal combination, which consists of a large distributed current loop with its axis in the transverse plane.

Conclusions

We present a formalism to calculate SNR and SAR with a homogeneous spherical sample, for any surface coil geometry as well as in the ultimate intrinsic case. Our method allows for quick simulations of the physical behavior of the RF field particularly in head imaging applications and provides ideal current patterns that can be used as a reference in coil design for parallel imaging and for parallel transmission.

References

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